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HOW TO CHOOSE A TEXTBOOK ON MATHEMATICS?

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Abstract: Creating of this article was motivated by the change of the new situation in connection with the Hungarian mathematics textbooks. Nowadays the teachers of Hungary could choose among the textbooks without restriction. The number of the textbooks is growing more and more. But our mathematics teachers have not any didactical support for the choice. Everybody knows that a suitable choice is not possible without an objective and measurable list of factors. We tried to give the Hungarian mathematics teachers such a list of factors, which is based on a theory concerning the choice of the English textbooks. This list of factors should make easier the choice of the textbooks on mathematics.

Zusammenfassung: Die Erstellung dieses Artikels war motiviert durch die neue, geänderte Situation in Verbindung mit den ungarischen Mathematik-Textbüchern. Heutzutage können die Lehrer in Ungarn ohne Einschränkung unter verschiedenen Textbüchern wählen. Die Anzahl der Textbücher wird immer größer. Aber unsere Mathematiklehrer haben in der didaktischen Unterstützung keine Wahl. Jeder weiß dass eine passende Wahl nicht möglich ist ohne ein Ziel und eine messbare Liste an Faktoren zu haben. Wir haben versucht, ungarischen Mathematiklehrern eine solche Faktorenliste zu geben, die auf einer Theorie zur Wahl der englischen Textbücher begründet ist. Die Liste der Faktoren sollte die Wahl der Mathematiktextbücher erleichtern.

Keywords: curriculum, problem-solving, selection, task, textbook

I. Analysis of the present situation

Until a few years ago, it was basically impossible to choose a textbook on mathematics in Hungary. The market of textbooks used to be dominated by a few series of textbooks for years 1-4 and 5-8, whereas for years 9-12 – due to the disintegration of the integrate education typical of the lower years – by 4. One series was prepared for students in vocational training, one for those in technical schools and two for the highest standard grammar-school education. One of these two textbooks is very elaborate and precise, and the other can primarily be characterized by easy comprehensibility.

Since the middle of the 1980s the efficiency of the mathematics education has decreased in Hungary, for several reasons. The phenomenon directly experienced by teachers of mathematics in practice has also been confirmed by national (MONITOR) [9] as well as international (PISA) [10] tests. By 2000 it had become obvious for everybody in education that it was time for a complete change in the situation. More modern content, together with its more modern treatment and greater practical applicability were necessary. Suiting the newly introduced so-called Kerettanterv (hereafter: Frame curriculum) a number of textbooks, series of textbooks with diverse concepts were published on mathematics. Although the possibility of choice is given, the methodology of choice has not yet been developed. How should a teacher choose from among the textbooks to achieve his/her aims with its help to the full? What is a good textbook like? We wish to make this decision easier for teachers by drawing up a list of aspects that helps them in choosing a textbook.

II. Theoretical background

The change in the market of textbooks has created a new situation. It was a situation completely unknown in the teaching of mathematics that in technical schools, for example, seven alternative textbooks were available instead of two. Such a phenomenon could only be witnessed in a single field of education earlier, namely in the world of textbooks on foreign languages, especially on English. Therefore we rely on this in our study. We thought that the wide range of the English course books provides us with the greatest experience for choosing the adequate series of textbooks on mathematics. Our aims were best characterized by the system described by Daoud and Celce-Murcia [3]. According to them, the four consecutive steps of textbook selection are as follows:

1. preliminary data collection,
2. survey,
3. analysis,
4. evaluation.

1. Factors that determine preliminary data collection are:

a) students (age, level, aim, general knowledge),
b) curriculum (social expectations, improvement of skills and abilities, practical application),
c) institutional conditions (size of class, number of lessons per week).

2. Survey means the overview of the related textbooks based on first impressions (in a bookshop). Glancing through the textbook in question, we can get information about the aims, methods and basic principles in it. This is when our first impressions are formed about the book. At this stage the analysis of textbooks is facilitated by the so-called catalyst-test [4]. According to the test the following questions must be raised concerning textbooks.

Communicative? Aims? Teachable? Available Add-ons? Level? Your impression? Student interest? Tried and Tested?

The initials of the words above make up the word "catalyst". According to Grant the function of a textbook in a class should be similar to that of a catalyst in a chemical laboratory: it should trigger changes [4]. The survey is based on the number of positive answers given to the questions in the test. The subsequent evaluation should only be performed for books that meet most of the expectations.

3. During the evaluation the textbooks that remain following the survey stage must be studied based on detailed lists of factors. The list of factors compiled on the basis of the work by Daoud-Celce-Murcia [3] and Cunningsworth [1] consists of six main parts.

- Aims
- Content
- Language content
- Skills
- Appearance
- Practical considerations

This list of factors – except for the language content – can also be applied for the criteria of textbooks on mathematics. Namely:

The part aims includes the examination of whether the textbook meets the requirements of society, the education program of the institution, the needs of the students, moreover the study of how comprehensive, detailed and flexible the textbook is.

As for the content, we have to make sure that the textbook satisfies the requirements based on its structure, function and topics, as well as on the abilities and skills to be improved. Are the exercises diversified and motivating? Is the explanation logical, precise and at the same time understandable? To what extent is the knowledge in it applicable in practice?

Language content is also important in the teaching of mathematics, although with emphasis different from that in foreign language education. It is the simple, clear, concise and precise wording of definitions, theorems, proofs or other rules that is significant in mathematics.

Skills involves specific aims like the improvement of meaningful, analysing reading comprehension, counting skills, orientation in space, wish for proving, pursuit of combinatorial argumentation etc.

Appearance is also essentially important as a motivating factor. Adequate figures and illustration, well-arranged paragraphs and readability all facilitate comprehension and thereby the acquisition of knowledge.

Practical considerations include the overview of the supplementary materials (teacher's book, exercise book, test sheets, sample solutions, tools) that belong to the given series of textbook in addition to the textbook itself. The durability of the book, its suitable size or acceptable price etc. are also factors to consider.

4. In the course of evaluation tests are prepared about the list of factors described under point 3, and then the series of textbooks are classified based on the answers given. Having obtained the scores, we are likely to give a correct answer to the question Which textbook should I choose?

III. Hypotheses

Prior to working out the list of factors related to choosing the textbook on mathematics we wanted to ask teachers of mathematics for their opinion. To compile a set of questions we needed preliminary hypotheses around which our study could be centred. In our opinion

(1.) the teaching of mathematics requires textbooks;
(2.) textbooks must primarily be written for students;
(3.) textbooks must teach problem solving.

The composition of our hypotheses was also facilitated by the opinions of students who had participated in a textbook-experiment organized by us [2]. Some of these opinions are below:

-„I like the textbook especially because there are sample problems solved with explanation."
-„… I am not too good at science subjects, but thanks to the explanations and precise solutions learning has become easier."
-„… another problem is that there are few problems to solve as practice."
-„The new textbook on mathematics is much better than the earlier ones. The problems get more and more difficult, the definitions are easy to understand."
-„I like the solutions of the problems, the fact that we can see various ways of solving a problem can give us ideas."

The teaching of mathematics in our schools is rather varied. We think that the answers to our questions are going to mirror this, just like the student opinions above.

Let us study these four points in detail.

(1.) A textbook is one from among many things that a teacher can rely on when creating efficient lessons [6]. The teaching of mathematics in our schools is rather varied, since there are no compulsory teaching methods. We think, however, that the answers to our questions are going to confirm our hypotheses, and the student opinions above also refer to this.

(2.) A textbook on mathematics may be intended for students or teachers, but the golden mean may also be experimented with. Our preliminary opinion is that a textbook should primarily address students. The problems, exercises motivate the students most if their own world, the things they are interested in appear in them, too. In the lower years it is important for a book to be colourful and spectacular. In year 12, however, cartoon figures would surely be absurd. Such funny illustrations might suggest that mathematics is an easy, readable and playful subject, nonetheless, its contrary is true. (There are some who disagree: textbooks with a concept opposite of what has been described can also be found in the Hungarian market [8].)

Clearer comprehension is facilitated by the sample solutions of problems, which allows an adequate preparation of the school-work. For us – because of the falling standard mentioned above – the inductive method seems to be correct. The basis of human thinking is also the system of inductive reasoning, from which deductive reasoning may later develop. We share the views of Skemp, who says that with definitions we cannot transmit anybody concepts of a higher rank than that of those known by them, but only by presenting a wide range of suitable problems [7]. Hopefully teachers will confirm this notion.

(3.) In the last two decades in Hungary, the problems of the final exam in mathematics at the end of the grammar-school period were chosen from one single book, which is entitled "Összefoglaló feladatgyűjtemény matematikából" (Exercises and problems in mathematics for classes I-IV). Although this book includes a considerable (4 193) number of problems, a lot of them, even whole units were ignored in

the course of time due to constant change (decrease) in requirements. This situation has resulted in the fact that teachers had to make students remember certain situations and tricks, practising control-type problems of the above mentioned textbook instead of the contents of logically compiled textbooks and their introductory, practice and control problems. This might have contributed to the disadvantageous educational processes. This situation has by now changed: for two years the problems of the final exam have not been selected from among those included in books available in bookshops. This might increase the role of both textbooks and the teaching of problem solving reasoning.

IV. The questionnaire

We distributed the questionnaires prepared by ourselves at a conference for teachers of mathematics. (The four-day long, accredited training was entitled "Tanítsuk eredményesen a matematikát!" (Let's teach mathematics effectively!). During the training mathematical, methodological, educational and psychological topics were discussed.) The questionnaire given to teachers included an introduction of a few lines. In this we informed the participants that the experiment is aimed at the development of a textbook-evaluation system. Participation was voluntary and anonymous. Participants only had to indicate whether they teach in years 1-4, 5-8 or 9-12 (that is in the junior, senior section, or at a secondary school in the Hungarian education system). Answers had to be represented by a number between 1 and 5. (5 represents the most correct, 1 the least correct answer.) 41 from the 45 teachers in the training returned a questionnaire appropriate for evaluation. We are fully aware that this number of questionnaires is not sufficient for a representative study, but it suffices to check the adequacy of the main ideas for establishing a theory.

To make the questionnaire more systematic and easier to evaluate, the 39 questions were classified into 5 bigger groups based on their orientation. Some questions were put up again in another group, too. Our purpose with this was to observe possible changes in opinions as well as to the reliability of filling-in, i.e. of the test. The construction of the questionnaire distributed among teachers was as follows.

I. Is a textbook necessary for teaching mathematics?

1. A textbook is necessary for studying mathematics.
2. A good textbook is primarily made for students.
3. A good textbook is primarily made for teachers.
4. A textbook is made for teachers, students and parents (assistant) alike.
5. Textbooks should regularly be used during lessons.
6. A textbook should be supplemented by a collection of exercises, too.
7. A textbook should be supplemented by chapter test sheets, too.
8. Instead of a textbook, a series of textbooks is necessary (textbook, book of solutions, test sheets, collection of exercises, teacher's book).

II. Usability

1. A good textbook is primarily made for students.
2. New information should be introduced with sample solutions of exercises.
3. New information should be discovered by students.
4. New information should be acquirable individually with a textbook.
5. A textbook should also suit teacher-aided acquisition.
6. The mathematical technical language should be interpretable for students.
7. No long, complex sentences should be included in a textbook.
8. It is readable style that is the most important with respect to wording.
9. It is precise mathematical terminology that is the most important with respect to phrasing.
10. Problems should be ordered according to the degree of difficulty.
11. The results of the problems in a textbook should be included in it.
12. The solutions of the problems should always be checked in a textbook.

III. Structure, content, requirements

1. All textbooks should include additional information.
2. Additional information should clearly be separated from compulsory information.
3. Textbooks on mathematics should not only include the requirements of the Frame curriculum.

4. Textbooks should not only convey technical information.
5. A textbook on mathematics should correspond to other subjects.
6. The information in a textbook on mathematics should be related to reality.
7. At the end of the chapters of a textbook a control work-sheet should be found.
8. At the end of a textbook there should be a summary.

IV. Expectations

1. A textbook on mathematics should educate.
2. A textbook on mathematics should teach problem solving.
3. A textbook should make students practise problem solving methods.
4. The teaching of correct interpretation of texts is also a task of mathematics.
5. A textbook should include memory improving exercises.
6. The problems in a textbook should improve the calculation skills of students.
7. Theorems should always be proved.

V. Design

1. A textbook should be bound in boards.
2. A textbook on mathematics should include a lot of illustration.
3. Essential information in a textbook should be emphasized in colours.
4. The more colourful a textbook is the more usable it is.

V. Evaluation

When analysing the results obtained, the average between 1 and 5 is always indicated. The value of the (corrected experimental) deviation falls most frequently between 0.7 and 1.2. It is only cases different form this that we discuss here. Based on subsequent consultations with teachers we can agree that in the case of an average above 4 the teachers agreed with the statements, in the case of an average between 3 and 4 opinions were divided, and a value below 3 means a definite refusal. We used this interpretation later in the evaluation.

The answers received definitely confirmed that textbooks are needed in the teaching of mathematics (based on I./1., that is point 1 of chapter I of the questionnaire). The average here was 4.439. It was interesting to see that while the rating (4.2) of teachers teaching in years 9-12 was surpassed by that (4.75) of those teaching in years 5-8, the teachers teaching in years 1-4 produced a value (4) lower than both of the previous two groups. During the interviews following the evaluation of the questionnaire it turned out that according to those teaching the youngest and the oldest groups of students it is the teacher and not the textbook that plays the main role in the acquisition of skills.

On the basis of the above it can unambiguously stated that hypothesis (1) holds.

In what follows it becomes clear that a good textbook takes the demands of both teachers and students (and their relatives) into consideration (I./4). This point received an average of 4.39. The case when a textbook is written for students (I./2.) is worse than this, its average is 3.854, the worst case is, however, when a textbook is written for teachers (I./3.). The value for this point is 2.439. Although it is not what we wanted to find, we expected this result. To make sure that the result was not influenced by the way of phrasing the question, in the following point the question was repeated – in another context. This idea changed the result.

We have already seen that textbooks are needed in the learning process (I./1.), but it has also become obvious that textbooks are not the only tools of school-work (I./5.) The 3.463 average – in accordance with the result of I./1. – was modified upwards by the senior section teachers (average 3.95), and downwards by the secondary school and junior section teachers (with values 3.133 and 2.667 respectively). The answer given to the two similar questions mirrored almost the very same attitude.

The same tendency can be observed in I./6., where the average is 3.902. The answers of the senior section teachers (average 4.4) are relatively easy to explain: contrary to those for the other years, each textbook used here is supplemented by a collection of exercises. The fact that our colleagues think that this is important

confirms the view that the other two fields would also need the compilation of books of exercises in harmony with the textbooks (in structure, orientation, wording etc.).

The very same explanation can be repeated for I./7., too. The average 3.39 also includes the 4.05 result of the senior section. (Here the teachers of mathematics are also provided with chapter test sheets.) In the case of secondary school teachers (average 2.6), however, the creation of such test sheets has only been attempted. The deviation results (1.221 and 1.262) show the deviation of the answers to I./6. and I./7.

The 3.976 average of I./8. summarizes and justifies the above very well. The 4.4 value of the senior teachers is the highest, and that of the others mirrors the view seen so far.

The questions of group II suggest how important it is to consider the students' needs in textbook preparation. Thus it is natural that the answer to question II./1., which is identical with I./2., gives a value (average 4.268) much higher than that of the first one. This fact showed us clearly that having reconsidered their points of view the teachers admitted that a textbook should primarily aim at students rather than teachers.

Based on the point studied we can state that hypothesis (2.) holds.

Further answers in part II proved to be in accordance with our expectations. On the basis of II./2. it is important for the teachers that new information should be introduced with sample problems (average 4.366). It is also significant that new information should be discovered by students themselves (average 4.195). True that this is somewhat less important for teachers of the youngest and the oldest groups of students; the average of both groups is 4. In the teachers' opinion older students may be able to use deductive methods that shorten the teaching time, due to their age and preparedness, while the youngest ones need direct instructions from teachers. This problem was further divided by the result of II./4. (average 4.024). The secondary school and senior section teachers agreed that new information should be acquirable individually with the textbook (averages 4.267 and 4.3). Teachers of the junior section find their pupils incapable of this activity (average 2.5). Considering the above it becomes clear that it was agreed that the textbook should also suit teacher's control (average 4.415).

As a result of PISA tests, greater attention has been paid to the problems with reading comprehension than earlier. This notion has apparently been accepted by teachers due to the media and the Ministry of Education. The numbers (average 4.976 and deviation 0.156) indicate that our teachers lined up with one another with respect to the interpretability of technical language in their answers to point II./6.

Naturally, traditions also have a role in phrasing mathematical concepts or theorems. It could have lasted for a long time, even for centuries for the wording of some notions to be formed as they are today. The world today, however, is changing very quickly. Something that used to be acceptable might need alteration in today's world of mass education. Such interpretation influencing factor is the length of sentences. In their answers to the statements in II./7. the teachers basically agreed that no long, complex sentences should be included in the textbook (average 4.537). For teachers in the junior section this statement is self-evident (average 5).

The question of readable mathematical texts depends on age based on II./8. It has the greatest importance in junior section (average 4.5), followed by the senior section (average 4.2), but it is not centrally significant in secondary schools (average 3.533). This result, or to be more precise this tendency could in fact have been expected.

The meaning of the sentence in II./9. is basically the opposite of that of the previous one. Therefore we thought that the smallest values should be expected from the teachers in the junior section, and the greatest average from those teaching older students. Well, this is not exactly what happened. The result of secondary school teachers was between the results of the other two groups. Having interviewed them we found out the reason for this. In general, the students of a secondary school are homogeneous, and at the same level with respect to their abilities. The good are capable of acquiring precise mathematical terminology, nonetheless for the poor it remains a mystery for ever.

If a textbook is primarily meant for students for individual use, without teacher's aid, then the problems in it must obviously be ordered according to the degree of difficulty: II./10. The teachers definitely agreed on this (average 4.561, deviation 0.673). At the same time, the score of the teachers of junior section pupils – in harmony with their opinion questioning the individual learning abilities of this age group – was lower than the other scores (average 4.167).

The statement in II./11. divided the teachers (average 3.049, deviation 1.359). Secondary school teachers agree that the results of the problems in textbooks should be included in the textbook (average 3.933), while the others definitely oppose this idea. Younger children would start the solution of a problem by looking up its result instead of trying to work it out themselves as usual – say the teachers.

The purpose of II./12. is to get information about the role of checking the exercises, that is solved problems in textbooks. The results are rather diverse (average 3.707, deviation 1.327). For the teachers in years 1-8 this is important, while teachers who train more advanced students this mechanical step is often omissible. It should compulsorily be performed only if it has some special function. The members of the other group are, of course, also right from their own point of view. In the early stages of teaching and learning mathematics, when basic knowledge and operations are not yet automatic, the precise and detailed description of repeated steps have a special role.

Section III is centred around questions related to the structure and editing of textbooks.

We think that for the sake of talent improvement and the attraction of attention, historical curiosities and other additional information besides the compulsory requirements must by all means be included in textbooks. The teachers basically approved this idea outlined in III./1. (average 4.146). We had expected a value slightly greater than this but accepted that for an appropriate efficiency of education teachers need a long time. It is curiosities that the enormous quantity of compulsory knowledge often takes time from.

Part III./2. rests on the previous point. Here the question is if there is additional information in a textbook should it clearly be separated from compulsory information. This was – maybe naturally and favourably – agreed basically by everybody (average 4.512).

Statement III./3. was intended to check our study. It is in fact almost identical to point III./1. The difference is that the question here addressed the relation of the textbook to the Frames of curriculum which essentially determine education. The result proved to be a justification of our expectation. The two average values were practically equal. (Here it was 4.1222.)

Let us proceed one step further now and assume that our textbook includes information also in addition to the Frame curriculum. Then we have two options: we may include technical and also non-technical additional information. Point III./4. tests the role of the latter type. This was least supported by the junior section teachers (their average is 3.33), on the other hand it is most agreed with by the secondary school teachers (their average is 4.267). If we consider that mathematics as a subject loses its popularity in direct proportion with the age of students, then the result becomes easily understandable.

In today's world the subject mathematics must not remain isolated but it has to make its applicability obvious for everyone. One of the most straight forward ways the students can understand this is the relation of mathematics to other subjects; this is what is referred to by III./5. Based on the value received (average 4.195) it can be stated that the participants support this idea. This is what we had expected. In our textbooks several (physical, chemistry, geographical, biological etc.) interrelations can be found.

As a result of the PISA test, it is becoming a more and more pronounced requirement for schools to convey realistic and applicable information: III./6. This has uniformly been accepted by the society of teachers as well as the ministry and the general public (average 4.732, deviation 0.549).

As was outlined above, our opinion is that textbooks must primarily be written for students. If the textbooks, however, are also suitable for individual study then the students must adequately be informed about their proficiency. One way of fulfilling this requirement is, for instance, to include control work-sheets at the end

of the textbook: III./7. (This is also what we have done.) The teachers agreed with this method (average 4.512, deviation 0.675).

The Hungarian curriculum has traditionally (for decades) allowed time for an end-of-the-year revision. As opposed to this, our textbooks have – also traditionally – been ignoring this. With III./8. we intended to find out the teachers' opinion on this subject. Well, the result has astonished even us. According to the teachers, such revision and summary chapters are badly needed in textbooks (average 4.488). On the basis of this we also have to reconsider the strategy followed so far.

The sentences in part IV are about expectations related to textbooks.

In the process of teaching education must not be separated from teaching. The education of a person requires the interaction of teachers and students. There are some subjects, however, in the textbooks of which educational texts may be included. We wondered whether mathematics is such a subject or not in the opinion of teachers (IV./1.). The answers made it clear that what teachers mainly expect from a textbook is precise, exact and professional description. Emotive or educating texts are held strange in a textbook on mathematics by most participants (average 3.634).

In Hungary there are many supporters of the Pólya method. His own teachers may also have used this – even if not as a conscious system of the method. Mathematics as such is especially suitable for improving problem solving skills. This seems to be justified by the fact that psychology also relies heavily on mathematical tools during experiments related to problem solving. It is another question if the textbook itself should teach problem solving (IV./2.). In the teachers' almost unanimous opinion it should (average 4.805, deviation 0.459).

This opinion clearly justifies the fulfilment of hypothesis (3.).

In addition to problem solving, another important field of the teaching of mathematics is to make students practise ways and methods of solving mathematical problems. This is primarily the task of teachers and not textbooks (average 4.171) – according to many in their answers to IV./3. The division of opinions is not even. The teachers in years 1-8 think that this could definitely be useful for younger students. Manifold assertion (by teachers, by textbooks) may facilitate the acquisition of basic techniques for solving problems.

The PISA test mentioned several times called attention to the fact that mathematics must convey applicable knowledge. Problems in real life arise, however, in different contexts. Therefore the teaching and improving of reading comprehension is also a task of mathematics (IV./4.). The teachers of mathematics accepted this idea (average 4.707, deviation 0.559).

Mathematics can be used for improving various skills. In addition to problem solving – although it is not usual in general – memory improving exercises can sometimes be included (IV./5.) This was supported by the junior section teachers the most (average 4.833), and by the teachers in years 9-12 the least, since such skills of their students had already been formed earlier. This can and must be improved at a younger age (average 4.293).

The question of improving the calculation skills (IV./6.) is similar to that of the previous part. So is the result (average 4.439, deviation 0.634). The slightly greater value of the average derives from the fact that this is supported by secondary school teachers to a greater extent. It is clear: memory plays little role in problem solving (at final exams the students are allowed to use a book of formulas), on the other hand poor calculation skills may have a really bad effect on the result.

Improving mathematical skills cannot be conceived without making students aware of the importance of proving. The proving of theorems was excluded from the requirements of the intermediate level final exam in mathematics in 2005. We wished to find out if the teachers agreed with this step. In IV./7. we asked if theorems always had to be proved. According to the teachers it is not necessary (average 3.61), thus the decision of the ministry seems to be subsequently justified.

In part V we surveyed a few formal criteria related to design.

In Hungary it is usually the textbooks (books of collected exercises or formulas, atlases) that are used for several years that are bound in boards. Even soft-covered books can hold 1 year. Up to a few years ago our publisher, the Műszaki Könyvkiadó (Technical Publisher) published books with both types of covers. The market unambiguously indicated that there was no need for costly books. Independently of this we wanted to know the opinion of teachers on this question (V./1.). The study justified the publisher (average 2.878, deviation 1.364). Although the greater deviation suggests that in high-standard schools the students with wealthy parents could easily afford to buy expensive textbooks, their number is insufficient for making the printing profitable.

Certain subjects have rather diverse features. While the acquisition of the contents of a book on biology, physics or the history of art is unimaginable without illustrations, a teacher of mathematics can very well do without these. There are few mathematical topics the illustrations of which play a meaningful rather than a mere aesthetic role. The question is whether a textbook on mathematics should include a lot of illustration (V./2.). The teachers do not think they should (average 3.634). It is enough for them to see numerous figures in the book that illustrate the texts or solutions of problems.

Although the role of illustrations in a textbook on mathematics is not too significant, it still might be favourable to include various typographic elements in a textbook. In our opinion, for example, essential information in a textbook should be emphasized in colours (V./3.). The teachers support this idea, too (average 4.512, deviation 0.637).

In Hungary several types of textbooks are available at present which are printed using 2, 3 or 4 colours. It is known that colours, colourful design can have an effect on emotions and experience. It is not known, however, if colours enhance usability (V./4.). According to the teachers they do not (average 2.829).

VI. An additional test

We had a chance to ask 20 teachers of mathematics of Hungarian nationality from beyond this country to fill in the questionnaire specified in part IV. We intended to find out if in our study there are specific factors which are valid typically under Hungarian circumstances. As we were mainly interested in the differences, we considered only the average values calculated from the total number of participants. We considered a difference significant – because of the difference in the division of those participating in the study (years 1-4, 5-8, 9-12) – if the difference of the totalized averages exceeded 0.5. Hereunder we mention only these cases.

As it turned out in the course of the subsequent individual interviews, the teachers from abroad gave extra scores to the collections of mathematical problems that belong to textbooks (I./6.), because in their textbooks there are not more than one hundred problems with sample solutions or one thousand appointed problems. Therefore collections of problems set up for textbooks mean great help for them (average 4.45).

Instead of the heuristic teaching of mathematics built on problem solving that is based on the principles of György Pólya several neighbouring countries emphasize the receptive, reproductive type of education. The lexical knowledge of their students is greater as the teaching there is not very student-centred. Thus for them good textbooks should not necessarily be made for students either (II./1., the average of which is 3.45).

The teachers of mathematics and students of Hungarian nationality from beyond this country are usually in a poorer financial situation. It would be favourable for them if they did not have to buy a separate book with the appointed problems, but the results of these problems were included in the textbook (II./11., average 4.45).

In the teaching of mathematics based on mechanic principles full of practice, it is advisable to realize the solutions of problems taking the same steps each time. Hence checking must always be performed – even if it does not involve anything new. It is clear that the average here (II./12.) was greater than in the other case (average 4.4).

In Hungary, the introduction of the Frame curriculum and later that of the Nemzeti Alaptanterv (National Curriculum) was preceded by long technical (and political) disputes. After this the system of the two-level final exams was developed, which gave new meaning to the concept of additional information. As has

already been mentioned, the problems of the final exam in mathematics were chosen from one single book, Exercises and problems in mathematics for classes I-IV for decades. Although every textbook presented the solutions of the most important problem types, the books were (naturally) different from one another. Two textbooks were prepared for students in grammar-school education, one for those in technical schools and one for those in vocational training. The textbook writers included everything in their books that they considered important. This knowledge specified by the book served as the basis for the mathematics school-work of the given type of school. Based on the previous section it becomes clear why a part of the participants from abroad could not interpret the concepts of additional information (III./2.) and the Frame curriculum (III./3.). Their role is less in another educational environment (average 4, and 3.5).

We already mentioned earlier that in some neighbouring countries the expansion of lexical knowledge has been aimed at instead of the improvement of problem solving skills. So, in these countries it is not very important for a textbook on mathematics to teach problem solving (IV./2., average 4.25).

VII. The list of factors

Having summarized all the experience described in the previous two sections, now we can attempt to draw up the steps of textbook selection in mathematics.

1. Preliminary data collection

During preliminary data collection, the following must be considered before taking the textbooks in hand.

a) students

What kind of mental abilities (attention, comprehension, memory, problem solving) do the students possess who the textbook is to be chosen for and how proficient are they? What are their purposes, how do they intend to use mathematics?

b) curriculum

What kind of abilities and skills do we wish to improve? What type of logical orientation (inductive-deductive teaching) can produce the best results?

c) institutional conditions

What is the size of the class, is the class divided in mathematics lessons, how many mathematics lessons are there a week?

2. Survey

Hungarian textbooks undergo several procedures in the course of the licensing process. The independent experts of the Ministry of Education examine if

- the book handed in meets the requirements of the Frame curriculum;

- the book is free from technical errors;

- the book is teachable from educational and didactic aspects.

Therefore the survey in a bookshop has to consider other points of view.

a) level

The textbook is prepared for the average, the better, or for both types of students. (For secondary school students this means the preparation for the intermediate or the advanced level of final exam, or both.) This latter type can be recognized by the fact that there is a huge number of (saliently marked and well separated)

additional parts. It must be examined whether the level of the textbook suits the level specified during the preliminary data collection.

b) structure

The structure of a good textbook is consistent. If a textbook has an inductive structure, then the order of teaching theorems (or other statements requiring proving) is as follows:

experience ⇒ presumption ⇒ interrelation ⇒ proof ⇒ generalization

The order of teaching in the case of a deductive orientation:

stating a theorem ⇒ proof ⇒ studying special cases

It must be examined if the structure of the textbook suits our preliminary ideas.

c) available supplementary materials

It is easier for both teachers and students if the problems in the textbook have their solutions also there – if the number of problems in the book is less than needed then a separate collection of problems belongs to the textbook –, if there are tested work-sheets (the solution of which is available together with a scoring and evaluation guide), if there is a curriculum in accordance with the Frame curriculum for teaching with the help of the textbook. In addition to the tools mentioned here, further ideas can naturally be thought of such as computer programmes or mechanic systems of devices. The question is whether there are supplementary materials available together with the textbook.

d) testing

The testing of textbooks before publishing them guarantees teachableness. The teachers participating in the teaching test approve of this by name. Here it must be examined if the textbook to be tested has been tested or not.

Each point of the survey requires an answer of ‘yes’ or ‘no’. It is advisable to carry out the further tests with at most 3 or 4 books the number of positive answers to which exceeds that of the other books.

3. Analysis

The list of factors for evaluating language books described in the theoretical introduction of chapter II cannot be applied here, or only to a limited extent. The textbooks on mathematics are in general – as opposed to language books – not international. They are adjusted to the traditions, requirements of a country (or a region). With respect to aims, for example, it can be stated that the purposes of the textbook accepted during the licensing procedure are in accordance with those specified in the Frame curriculum, thus also with those of local curricula built on the Frame curriculum. Language content is also examined during the licensing of textbooks. The average length of sentences, the frequency of the longest sentences, the occurrence of foreign and technical vocabulary are all examined. Besides this, qualified experts count the figures, pictures in textbooks, as well as if their use is autotelic. Possibly these aspects must also be included among the factors in other countries. Now let us study the most important elements for us.

1. The textbook is appropriate for individual learning.

Demonstration: with control work-sheets. It is a natural demand of students to determine the efficiency of individual learning. Without this the whole learning process would not be complete. (The question if a textbook is appropriate for individual learning can be examined in several ways. Our aim was to find a factor for this that is very easy to determine.)

Scores	
Each unit is supplemented by control problems	2 points
There are control problems in the textbook	1 point

There are no control problems in the textbook 0 point

2. The textbook is well applicable in the teaching process.

Demonstration: by classifying the problems in the book according to the degree of difficulty. Classifying the problems in the book makes applicability easier for all participants of public education. (For teachers, who choose problems from the book; for students, who wishes to learn from the book individually, too; and for parents, who wants to get their child to solve another problem at home.)

Scores
Problems are divided at least into 3 groups 2 points
Problems are divided exactly into 2 groups 1 point
Problems are not divided into groups 0 point

3. The logical orientation of the textbook is inductive.

(This is how we can make sure that new information is introduced by sample problems, and that students should feel as if they discovered these themselves.)

Scores
The structure of the textbook is definitely inductive 2 points
The structure of the textbook is partly inductive 1 point
The structure of the textbook is not inductive 0 point

4. The textbook makes talent improvement possible.

(Textbooks must include all information needed for the acquisition of knowledge (specified by the Frame curriculum). Therefore no differentiation is possible among them in this respect. The differentiation that allows talent improvement is, however, an extra feature.)

Demonstration: with additional information in textbooks.

Scores
There is additional information in the textbook, and it is clearly separated from compulsory information 2 points
There is additional information in the textbook 1 point
There is no additional information in the textbook 0 point

5. There were supplementary materials prepared with the textbook (problem solving book, a collection of exercises, test sheets, curricula etc.)

Scores
There are at least 3 supplementary materials with the textbook 2 points
There are 1-2 supplementary materials with the textbook 1 point
There are no supplementary materials with the textbook 0 point

4. Evaluation

Evaluation means that we add the scores received during the analysis for each textbook. It is the textbook with the highest score that we can consider the winner of the list of factors outlined above. And this is what we can recommend for teachers and students.

VIII. Summary

The study was started by several factors. Such a determining factor was the summary article written by our fellow teacher of English about choosing an English textbook [5], and also that the world of the market of textbooks is not strange for us either: both of us are authors of some books and textbooks written for students. Our aim with the present article is to set of a list of factors for teachers of mathematics in Hungary which adequately prepares them for choosing a textbook on mathematics. It must be obvious for us that teachers cannot spend days examining textbooks in bookshops. (A precise, measurable and unambiguous list of factors is necessary for teachers in order to be able to carry out an objective study.) Therefore we had to

ignore a few factors (such as problem solving) that are difficult to examine. We are aware of the fact that the list of factors presented here is only one possible solution. We hope that our study may contribute to the initiation of a dialogue about textbook selection. At the same time, on the basis of the results obtained from groups of teachers from abroad, we are hopeful that after the necessary corrections and following further tests the method itself might prove to be adaptable by teachers of mathematics in other countries, too.

Appendices

Table 1. *Results of the main experiment*

	Mean	Deviation	Mean (1-4)	Mean (5-8)	Mean (9-12)
I./1.	4.439	0.838	4.0	4.75	4.2
I./2.	3.854	1.195	4.0	3.9	3.733
I./3.	2.439	1.026	2.5	2.65	2.133
I./4.	4.39	0.891	4.167	4.5	4.333
I./5.	3.463	1.075	2.667	3.95	3.133
I./6.	3.902	1.221	3.167	4.4	3.533
I./7.	3.39	1.262	3.167	4.05	2.6
I./8.	3.976	1.193	3.667	4.4	3.533
II./1.	4.268	0.923	4.167	4.5	4.0
II./2.	4.366	0.994	4.5	4.45	4.2
II./3.	4.195	0.843	4.0	4.4	4.0
II./4.	4.024	0.987	2.5	4.3	4.267
II./5	4.415	0.741	4.17	4.6	4.27
II./6.	4.976	0.156	5.0	5.0	4.933
II./7.	4.537	0.745	5.0	4.4	4.533
II./8.	4.0	0.88	4.5	4.2	3.533
II./9.	4.317	0.82	4.0	4.45	4.267
II./10.	4.561	0.673	4.167	4.65	4.6
II./11.	3.049	1.359	2.667	2.5	3.933
II./12.	3.707	1.327	4.0	4.05	3.133
III./1.	4.146	0.792	4.167	4.3	3.933
III./2.	4.512	0.779	4.667	4.45	4.667
III./3.	4.122	0.9	4.0	4.35	3.867
III./4.	3.951	0.947	3.333	3.9	4.267
III./5.	4.195	1.1	4.0	4.3	4.133
III./6.	4.732	0.549	5.0	4.8	4.533
III./7.	4.512	0.675	4.0	4.8	4.333
III./8.	4.488	0.84	4.5	4.85	4.0
IV./1.	3.634	1.135	2.833	4.25	3.133
IV./2.	4.805	0.459	4.667	4.85	4.8
IV./3.	4.171	0.863	4.333	4.4	3.8
IV./4.	4.707	0.559	4.833	4.65	4.733
IV./5.	4.293	0.75	4.833	4.4	3.933
IV./6.	4.439	0.634	4.833	4.45	4.267
IV./7.	3.61	1.181	4.333	3.65	3.267
V./1.	2.878	1.364	2.5	3.0	2.867
V./2.	3.634	0.994	3.667	3.5	3.8
V./3.	4.512	0.637	4.333	4.7	4.333
V./4.	2.829	1.16	2.667	2.8	2.933

Table 2. *Results of the additional test*

I./1.	4.9
I./2.	3.35
I./3.	2.8
I./4.	3.95
I./5.	3.5
I./6.	4.45

I./7.	4.1
I./8.	4.05
II./1.	3.45
II./2.	4.55
II./3.	4.35
II./4.	3.7
II./5.	4.35
II./6.	4.95
II./7.	4.6
II./8.	3.7
II./9.	4.55
II./10.	4.85
II./11.	4.45
II./12.	4.4
III./1.	4.0
III./2.	4.0
III./3.	3.5
III./4.	3.8
III./5.	4.4
III./6.	4.65
III./7.	4.75
III./8.	4.35
IV./1.	3.85
IV./2.	4.25
IV./3.	3.9
IV./4.	4.75
IV./5.	4.5
IV./6	4.4
IV./7.	3.3
V./1.	2.8
V./2.	4.05
V./3.	4.4
V./4.	2.55

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