



COMPUTER ALGEBRA, VIRTUAL LEARNING ENVIRONMENT AND MEANINGFUL LEARNING: IS IT POSSIBLE?

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Abstract: A major challenge faced by teachers nowadays relates to the usage of proper educational technology to achieve a true and meaningful learning experience involving time for reflection. Teachers constantly seek new ways to improve instruction, but in virtual learning environments they often find themselves in a new role, interacting in a dynamic system with students and simultaneously acquiring new skills related to the tools in use. In this paper we address this question by conducting an online course aimed at primary and secondary school mathematics teachers, designed to investigate the effective use of GeoGebra and Moodle. The tools selected for the course are free and easy to use, an important factor for the new technology to be incorporated into teaching practice. The course results show that a well-constructed proposal is able to meet the expectations of teachers. Furthermore, the usage of new technologies involves, beyond technical issues, changes in the behavior and in the relationships between the actors involved.

Key words: Distance Learning, Teacher Education, Moodle, GeoGebra.

1. Introduction

New information technologies help to improve the design and production of resources and educational environments but often lead to a permanent demand for professional updating. Online courses can address these needs producing results beyond expectations.

Effective participation in online courses provides new opportunities for teachers, integrating them truly in the globalized world. However, the usage of virtual learning environments and educational software can be a challenge for teachers who may feel as beginners among young students grown up with the Net Generation technology.

In fact, when facing telematic resources, teachers may find many obstacles from technical and non-technical nature. At the same time that new technologies increase possibilities of developing teaching projects, deeper understanding of specific contents and better knowledge of educational technologies are required.

One needs to revise concepts, identifying the kind of education to be achieved and understanding the new role of teachers and their different forms of interaction in this new scenario. The integration of resources offered by technology to achieve a truly and effective learning experience is the great challenge.

As part of our research project we offer the course "Dynamic Mathematics," where participants experience an online course in new geometry and algebra teaching techniques. Furthermore, we want to give participants the opportunity to reflect on important educational issues and to develop new skills for research.

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In this work we choose two computational systems, Moodle and GeoGebra, not only because of the facilities and features they offer, but mainly because they are appropriate to our goals. The first is a virtual learning environment that favors collaborative learning, enabling interactions with resources from different media and among all participants. The second allows the exploration of geometry and algebra in a dynamic and interactive way, thus becoming an important aid for investigation of conjectures and verification of mathematical properties.

The course is organized in thematic units in accordance with curricular guidelines for Primary and Secondary schools, allowing participants, mathematics teachers, to evaluate the scope of technology used and possibilities of adapting it to their needs.

The course is part of the research developed by the Digital Technologies in Mathematics Education (TecDEM) research group and is offered twice a year since 2009, for classes with a maximum of 20 (twenty) students, mathematics teachers from various regions of Brazil, through COGEAE – the General Coordinating Committee for Specialization, Improvement, and Extension, an academic unit at the Pontifical Catholic University of São Paulo - PUC/SP, which, since 1983, has continually developed educational activities organized and proposed by teachers and researchers in various fields of knowledge.

2. Methodology

The learning situation, in a distance learning course, is similar to messy situations in real life learning, in which there are several interruptions, and students cannot always focus on the task. Social interaction is complex and goes beyond the traditional classroom, involving all participants of the course, everything and everyone that is accessible on the Web.

The instructional design can and should be constantly reviewed, with the participation of students and teachers. In an online course this refinement can be done almost in real time. In this course, we try to improve teaching practice during the performance, making interventions we perceive to be necessary to meet the specific needs of participants.

We consider it appropriate to include two personal meetings, one on top one another at the end of the course to complement the analysis of the nature of students and some climate and systemic variables such as engagement, commitment, self-discipline, ease of adoption of a similar activity into the curriculum according to [5].

In a distance learning course, we have to choose the mechanisms available in the virtual environment that we consider appropriate for the qualitative evaluation of the learning variables. Thus, in this project, all students' interaction with the teacher or the environment are permeated or complemented by statements, discussions or comments provoked in some form of interview.

We adopt an approach to mathematics with eight thematic units, so that our teacher-student can understand the reach of technology and can design small pieces suitable for teaching their own classes.

While developing the activities, we assume that the following aspects are important:

- Enhance the role of the teacher in presenting issues and methods for problems solving;
- Propose experiences going beyond the level of activity proposed;
- Indicate texts related to different approaches to the same topic;
- Present the mathematical content so that some resources of GeoGebra are essential to solve a particular problem;
- Expose explicitly the use of GeoGebra by showing simulations, arguments and different registers.
- Be consistent with the theoretical and methodological assumptions of Mathematics Education.

The research methodology of this study can be considered a “design-research” [4] as it involves the instructional design and the research supported by activities in the classroom.

According to these authors, the design experiments aim to contribute to the development and comprehension of "learning ecology" i.e., of complex systems that involve multiple elements of distinct natures. The elements of a learning ecology typically include the tasks and problems that the learners will face, the tools and resources supplied for their resolutions, and the practical means by which the teachers can orchestrate the relationships between these elements in their classrooms. The use of the metaphor regarding ecology emphasizes the interactive nature of the contexts investigated and the importance of analyzing its various elements together rather than separately.

According to [6] this methodology:

[...] requires several analysis cycles to improve the product and the interpretation at multiple levels. [...] the gathering and interpretation of the data do not occur at the end of the experiment, but the collection per se during development and the interpretation of data in all the levels should generate and refine principles, properties, and products that are increasingly more useful to researchers, teachers, and other professionals (p. 117).

3. Design of the modules

The mechanisms we selected from Moodle - Forum, Assignment, Journal, Portfolio - are interactive activities and important part in the project. These activities allow students and teachers sharing ideas and productions during online experience. Static material, like Files, Web Page, Link are used to offer students contents of the course.

Each module contains static materials, a Forum, an Assignment and a Journal. The Portfolio is the resource used to share geometric GeoGebra constructions made by students to solve special problems.

At the Forum we present an article about some research on Mathematics Education, with educational content, involving the subject being studied at the module. We highlight the main points of research and formulate questions to encourage discussions about it.

The Assignment is being used not only for evaluating the competence acquired on GeoGebra, but also to obtain information about the choice of exercise, difficulties and satisfaction with task performance.

The Journal is the mechanism used to provoke the student's reflections on their own action. We consider that it is important to cultivate the habit of systematic reflection, to improve their investigative conduct as well as their professional development.

We choose topics that help develop and improve teaching practices of mathematics teachers, based on themes of recent research in mathematics education and we set the amount and depth of content to allow everyone to dive or stay on the surface, according to their needs. Thus, the selected topics are the following:

1. **FIRST STEPS:** In this module, the first steps in the fundamental constructions of Geometry and in the exploration of GeoGebra were taken. In this first part, it is necessary to acquire familiarity with the screen, with the basic resources of GeoGebra and in particular to investigate the fundamental constructions of geometric drawings. We choose to present the basic resources of the application in the midst of simple geometric construction problems.

The text [9] supporting the activities of this module point out the cognitive difficulties of students in geometry and the contributions that the dynamic geometry environments offer to overcome these difficulties.

2. **POLYGONS:** In this module we aim to study the polygons, providing for beginners in geometry interesting research. We started the module with the construction of quadrilaterals and triangles and recommend resource use GeoGebra to verify properties of geometric figures.

The text by [7] that was shared in the forum of this module deals with barriers to the effective use of technology in school, and with conceptions about its use in Mathematics Education.

3. **CIRCLES:** The properties of circles, regarded as intuitive, but not always so simple and the construction of mosaics based on the circles were the main topics of this module. We investigated the geometric properties of circles with respect to tangents, chords, arcs and angles plants.

The text [14] dealing with the different functions of the demonstration in Mathematics Education is important for this stage of the course. The role of technologies in mathematical demonstrations is a subject that is not studied in depth in the initial education of teachers.

4. ISOMETRIES: Isometries are revealed in nature [2], in almost everything that exists in the world of art, and in constructions made by man. In this module, we investigated the isometric transformations – translation, reflection, and rotation, as well as some coverings of the plane. In Figure 1, a sample activity.

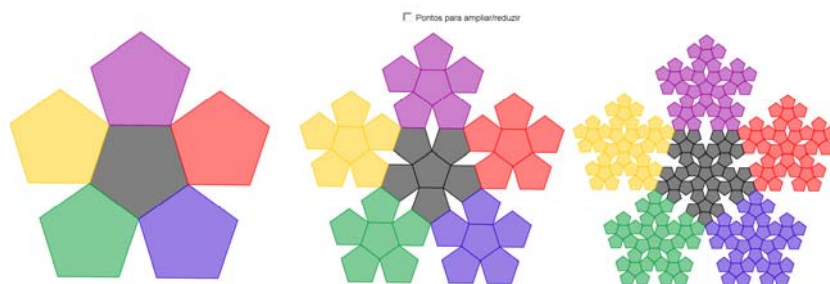


Figure 1. Sample Activity in Module4

The text [11] suggested in this module, discusses aspects of the relationship between Art and Mathematics, and its trajectory through time.

5. AND ALGEBRA?: In this module, we explored some algebraic resources of GeoGebra. The equations and coordinates can be entered directly in the command area and it is possible to manipulate variables associated with numbers, vectors, and points. The dual geometric and algebraic presentation of the objects studied in GeoGebra enables the natural progression from synthetic geometry (based on the geometric properties) to analytical geometry (based on algebraic equations).

The article by [3] presents considerations about Algebra and Algebraic Education, triggered by responses to questions about the subject given by students taking courses for a Mathematics Teaching Degree. From these responses, the authors discuss concepts of Algebra and Algebraic Education, as well as about learning styles of learning, concluding that these subjects need to be discussed more deeply in primary and secondary school teacher education courses.

6. STUDY OF FUNCTIONS: In this module, we continue with the algebraic exploration of GeoGebra, working with functions, a fundamental concept in Mathematics, establishing definitions and properties, and exploring the different records for representation (natural, symbolic, and graphical language). Follow is a sample activity.

In this activity (Figure 2) we will explore equations and inequalities of a variable. The process for exploring these situations comes from applications of the definition by parts of absolute value. The absolute value of x is equal to x if x is positive; it is equal to $-x$ (negative x) if x is negative, and it is equal to 0 if x is equal to 0.

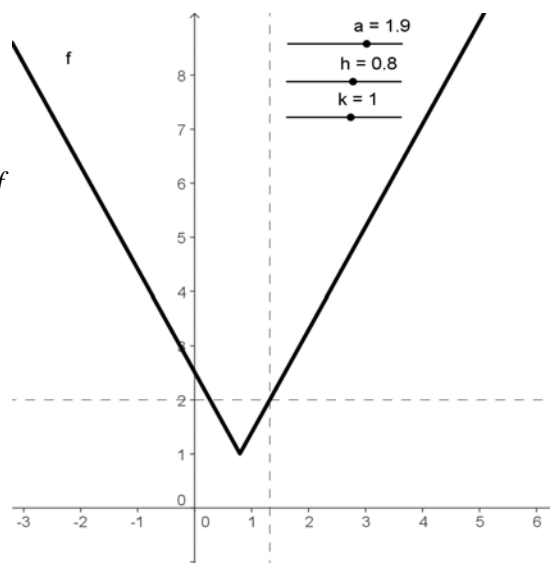


Figure 2. Sample Activity in Module6

In the text suggested in this module, the author, in [10], proposes the use of mathematical software in the discipline differential and integral calculus as a tool for problem solving and also provide research environments and generalization of concepts. He justified his proposals, noting that discipline allows the use of multiple representations and that we can take advantage of the computing environment. When it is possible, the teacher can encourage students to analyze problems in different contexts, in multiple representations.

7. TRIGONOMETRY: We note that in trigonometry are studied the relationship between sides and angles of a triangle and, that it is closely related to mathematics and astronomy. Situate the student within the context of the subject is very important because it shows that mathematics has not arisen by chance, began to be developed according to man's need and is a living science that is always under construction. The following is a sample activity.

In this activity (Figure 3) you will construct an animation that traces a special curve called a "sinusoid." Variations of the "sinusoid" curve are graphs of the functions that are called periodic functions; that is, functions that repeat. The movement of a pendulum or the waves of the ocean can be examples of periodic functions.

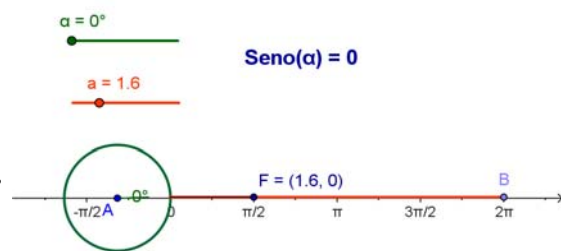


Figure 3. Sample Activity in Module6

The text [12] reports a classroom experiment, in the first year of high school, of a public school in the state of Paraná, Brazil. A motivating subject was used, bringing the contents of trigonometry with emphasis on the history of mathematics to assist in the motivation for learning. The authors consider that at present, it is essential to know the history of mathematics for a better understanding of the concepts and also to allow its use in the classroom, as a pedagogical resource.

8. LOCUS: In this module, by means of results involving mathematical objects, we offer an introduction to the concept of geometric place, exploring the facilities of GeoGebra for its construction.

The forum of this module discussed the text [1]. The authors raise questions about the use of computational resources for teaching and learning of geometry, trying to show the changes that occurred with the replacement of using paper and pencil by lines make with computational resources. From the results presented in the text, we sought to expand the discussion of informatics in education, primarily as it relates to geometric constructions and recovering the work developed throughout the "Dynamic Mathematics" course.

4. Conclusion

In this paper we present the development of an online course aimed at primary and secondary school mathematics teachers, designed to investigate the effective use of GeoGebra and Moodle, important tools for to be incorporated into teaching practice. The usage of new technologies involves changes in the behavior and in the relationships between the actors involved.

The course results show the importance of the experience for teachers who plan to adopt a new technology in their teaching practice. The comments from students show their involvement and commitment to learning. Some observations indicate the importance of estimating the time needed to perform activities and of periodical estimation reviews.

... the performance of exploration and study activities of the tutorial "Primeiros Passos" were essential for this first approach. In this sense, 6 hours provided were insufficient.

Self-discipline for a reflective study is highlighted by some participants:

... I was doing step by step with plenty of calm, following all the instructions ... because then I will have security to teach my students. I'm very involved with all these activities, at the beginning I was a little shaky, I had read several times, back to the beginning, but as I was doing it would seem simpler

The possibilities of adopting the activities in their teaching practice began to emerge at the time that some students brought the experiment to their own classrooms.

I started using some models with the students ... The benefits are many, the lesson becomes more attractive, and they well understand the definitions and their applications. We are progressing....All activities ... can be applied in the classroom.... I use a model made of acrylic,... while making the activity in this module, I was already thinking on its introduction as a tool for enhance and facilitate the understanding of the theorem... After the construction, beginning statements to better understand the concepts, questions arise during this period, interesting questions... Besides the use in mathematics, I am developing an activity with Geogebra which allows the verification of properties from image construction in plane and spherical mirrors ... I hope to finalize the activity until the end of this course to share it with other colleagues.

We also note a change of attitude towards the meaningful learning of teaching, as the statements reveal:

My expectations were exceeded because I thought it would be a course related only to carry out activities, but the texts and discussions in the forum have a great theoretical support, allowing for reflection on teaching practice.... The text that discussed the issue of the demonstration was very useful and has enabled the act to think about thinking.... The text enabled me to understand that some methods used in the classroom can easily reach some specific groups of students.

The subjects studied in each module were well received by the students and have generated new aspirations and expectations. On the other hand students were aware of the impossibility, due to the short time frame, of a deeper study of the presented subjects and the proposal of new ones as this would interfere in a meaningful reflection about their education and teaching practices.

My expectations were met beyond what I had hoped for; in fact, I had a different idea of what distance learning would be. It was "distance" learning only in name, since I had complete assistance whenever I needed it.... As a suggestion for improvement, as already mentioned in a prior diary text, I would like to insist on an extension of this course so that we can investigate, discuss, reflect, apply, and further leverage this tool.

The combined utilization of GeoGebra and Moodle proved to be very successful as the Moodle GeoGebra filter enables the inclusion of GeoGebra files directly into Moodle web pages and discussion forums. This allows the usage of GeoGebra applets to complement communication with students extending it beyond the written form. The videos and GeoGebra animated activities and applets embedded in Moodle were widely used and showed to be a valuable contribution to the learning process. The challenges of distance education have been overcome by the students and the course results show a possible path for continuing education. The statements consolidate the presented course proposal.

With each class and with each access to the Internet in the virtual learning environments, I grow and always learn something new. The physical presence meetings were excellent, since we exchanged some of our experiences and we could have a more personal contact,

eye to eye, with the participants. However, the Virtual Learning Environments are extremely rich. Through them, we greatly developed our creativity, our writing, our form of communication. It is an environment of mutual collaboration, where one learns with the other. We are all apprentices at some time, and collaborators at others. Each one of us has a new knowledge to be shared.

With the understanding that technology is just a mediator in the process of teaching and learning, the teacher must assume the role of the primary protagonist in the activities of mobilization, creativity, and exchange of experiences with their peers, so that positive changes may occur in their teaching practices.

Undoubtedly, the integration of activities with GeoGebra, readings and discussions of educational articles as the use of Web resources helped to create a meaningful learning environment. We note that the effective integration of technology in education involves issues beyond teachers commitment as school organization and support material are essential first steps for the usage of new technologies.

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