



A CRITICAL APPROACH TO SCHOOL MATHEMATICAL KNOWLEDGE: THE CASE OF “REALISTIC” PROBLEMS IN GREEK PRIMARY SCHOOL TEXTBOOKS FOR SEVEN-YEAR-OLD PUPILS

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Abstract: The reference contexts that accompany the “realistic” problems chosen for teaching mathematical concepts in the first school grades play a major educational role. However, choosing “realistic” problems in teaching is a complex process that must take into account various pedagogical, sociological and psychological parameters.

This paper presents a selection of problems from the new mathematics textbook of the 2nd grade of Greek primary school which were solved and commented upon by 27 postgraduate students.

The results of our research showed that the empirical reference contexts put forward in these problems create many difficulties in their “deciphering”, which leads us to conclude that their pedagogical role is problematic.

Key words: Mathematics education, early years, reference contexts.

1. Introduction

School mathematical knowledge is the product of a process of “didactical transposition” (Chevallard, 1992) or, in other words, processes of recontextualisation (Bernstein, 1990). In order to shape it, knowledge is selected from the scientific field of mathematics, and then simplified and adapted to the age of the pupils to whom it is addressed. The final shaping of the content of textbooks is also influenced by the choices made regarding pedagogical and didactic knowledge management. These emanate from the epistemological acknowledgements of official pedagogical discourse which, in every curriculum reform, are linked to specific theories of instruction (Morais, Neves and Fontinas, 1999).

According to the new primary school curricula which were implemented in 2006 and are in force to the present day, problem solving is one of the main objectives of mathematical education in Greek schools. Indeed, the solving of problems contained in textbooks may constitute an autonomous learning objective, aimed at activating children’s mental functions in order for them to develop the ability to identify connections and synthesise the parameters involved in a problem; or it may constitute a means by which to teach particular topics in school mathematics.

In any event, the interest of textbook authors is focused on the introduction of suitable problems, which will make sense to the children and will engage their interest by drawing on pre-existing knowledge and experiences (Siegel and Borasi, 1992). Moreover, in order for a problem to be manageable by a child, it should be on – or adjoining – the level of its cognitive capabilities. According to Vygotsky (1978), the problem should lie in the child’s zone of proximal development. Thus, the new information and new knowledge incorporated by mathematics problems should guide the pupil to enrich the way in which he/she thinks and works (Grugetti and Jaquet, 2005).

In the first school grades, mathematics problems are introduced through the use of suitable reference contexts. In particular, by utilizing these contexts, an effort is made so that the way in which

mathematics problems are presented engage children's interest and that, through the solving process, children are able to assimilate the school mathematics being taught.

In this paper, we examine a selection of problems from the 2nd grade textbook of Greek primary school which is intended for seven-year-old pupils, and we investigate aspects of the educational context that accompanies them, in order to assess the possibility of their pedagogical management.

2. Theoretical observations

The way in which pupils learn the meaning of mathematical concepts is related to the semiotics of teaching (Radford, 2006). In the context of this semiotic process, emphasis is placed on the central role of the cultural environment in the building of the content of each mathematical concept. This approach holds that mathematical ideas and concepts are conceptual forms of historically and culturally mediated human activity, and that the knowledge we possess, as well as the way in which we learn, are framed by ontological stances and cultural processes that impart a particular content to the problems posed in a mathematics lesson.

Regarding school mathematical knowledge, H. Steinbring (2005) claims that teaching mathematics is considered a specific cultural practice that aims at introducing school mathematical knowledge into the classroom culture, and that the appropriation and encoding of mathematical knowledge requires the development of appropriate *sign or symbol systems* (Steinbring, 2005, p. 21). The sign is a mathematical object, which, in the first school grades can be the arithmetical signs we are all familiar with. These signs refer to quantities; arithmetical sentences such as sums or products; tables; graphs; as well as verbal expressions. The mathematical importance of a sign lies in its function and the significance we attribute to it. The symbol contains all the characteristics and functions of the sign, while at the same time having its own, internal structure. For example, "the symbol €17.05 has such a structure, given to it by the decimal positional system, but the traffic sign STOP does not indicate such a structure" (Steinbring, 2005, p. 22). The objects and reference contexts used in the first school grades belong mainly to the class's educational material or are related to pupils' experiences. In the latter case, use is made of horizontal discourse elements, that is to say, elements that are drawn from the pupils' everyday world (Bernstein, 1999).

The relationships that develop in the context of mathematical education between signs and symbols on the one hand, and objects and references on the other, are not subjective or arbitrary, but are defined by epistemological conditions of organizing mathematical knowledge. In this way, a particular educational material or reference context are interpreted in a specific way in a mathematics lesson. Moreover, mathematical signs and symbols take on a specific content which is defined as much by the context within which the mathematical concept is processed, as by the pupils' pre-existing mental representations of the concept in question. According to Steinbring (2005, 2006), the relationships between objects/references, signs/symbols, and concepts, form an epistemological triangle which describes a basic structure of interaction within the classroom, which in turn is based on the building of mathematical knowledge. In this theoretical framework, signs and symbols are different from objects. Also, in order for new and unfamiliar signs and symbols to take on a specific mathematical content, they will have to be linked to specific objects and reference contexts. In a future stage of the pupil's forming a concept, the external representations and specific contexts used in introducing mathematical knowledge to the first school grades are gradually replaced by notional representations and abstract concepts.

The observations made above show the pedagogical importance of enriching the teaching process, especially in the first school grades, with suitable external representations that contribute to the building of mathematical concepts.

In this pedagogical perspective, the "realistic mathematics education" approach may prove both creative and effective, as long as we have appropriately chosen the educational material, which should be based on reference contexts familiar to the pupils (Van den Brink, 1989; Gravemeijer, 1997; Wubbels et al., 1997). The realistic approach to education is a mediation between the concrete and the abstract and places emphasis on the development of modelling capabilities on the part of the subjects who are learning. The principal difference between this approach and mechanistic and typical

approaches is that realistic mathematical education does not have as its basis abstract axioms and rules that aim at specific applications. Furthermore, it is not interested so much in the implemental aspect of knowledge, as in the process of building, by the children themselves, of the knowledge being sought. The viewpoint of “realistic mathematics” emphasises teaching situations (Gravemeijer, 1997) which are defined as the learning contexts within which atypical learning strategies develop and where the meaning given by the pupils is encompassed by intensely personal characteristics. The teaching situations must be planned carefully. At first, we focus on the kind of application to be used in education. In addition, we explore whether the proposed teaching situations facilitate achieving the goal of progressive mathematising. That is to say, whether they facilitate the effort to create in the pupils the suitable experiences which will lead them to build the mathematics they are being taught (Woubbels et al., 1997). In the course of mathematisation, despite the frequent vagueness and overlapping of their borders, we can distinguish two levels (Freudenthal, 1983): the first is horizontal mathematisation, in which a transition is attempted from the real world to the world of symbolic writing. Horizontal mathematisation practices are encountered mainly in the first school grades. The second level is vertical mathematisation. Here, mental processes are extricated from the concrete world and move on a level of abstract concepts and symbols.

The purpose of this research is to study special cases of “realistic” problems, whose communication framework appears difficult for pupils to “read”, and, more specifically, to explore possible difficulties that arise from the incompatibility of the “realistic” context and the mathematical treatment of the problems on the one hand, and the way in which this may influence the acquisition of new knowledge on the other.

The problems used were selected from the 2nd grade textbook of Greek primary school.

It should be noted that in the Greek educational system, mathematics textbooks are the same for all schools and are selected by the Pedagogical Institute. Moreover, according to the most recent educational reform regarding the teaching of mathematics in Greek schools (2003), mathematics and their teaching is strongly classified (Bernstein, 1990; Koustourakis and Zacharos, 2010) in the primary school curriculum, that is to say it constitutes a distinct cognitive subject.

3. Methodology

3.1. Sample

The research was carried out on twenty-seven postgraduate students (belonging to two postgraduate courses – thirteen and fourteen students from each course), the majority of whom were teachers of different school grades. Furthermore, those who were not working as teachers intended to become employed in primary and secondary education in the future.

3.2. Design

The students that made up the sample, who were attending a course on the research and didactic approaches to teaching mathematics, were asked to solve three “realistic” problems taken from the math textbook of the 2nd grade of Greek primary school.

The problems: The problems (Kargiotakis et al. 2006, *Student's Book*, vol. I, p. 45, see Figure 1) use drawings to describe a commercial transaction. The children shown are buying toys whose value is written on small cards. Also given are the amounts of change that the children received after paying for their toys. The problems, therefore, ask the pupils to calculate how much money each child gave the shop owner.

The problems belong to the textbook section that aims to familiarise pupils with the currency of countries within the Eurozone.

According to the typology introduced by Vergnaud (1979; 1983), these problems belong to the additive structures category, and specifically to the transformation of measurements of a quantity. In this case, an initial quantity (the money the child starts out with) is transformed (negative transformation), giving a final quantity (the change). In mathematical terms, this is expressed as: $a - b = c$ and pupils are asked to find quantity a . Additive problems of this kind are considered easy for the

pupils of the first grades of primary school, but they become more difficult when the unknown quantity is one of the addends (Vergnaud, 1979; 1983).

The problems were solved individually by those participating in the research and participants were asked to give their solutions in the way defined by the textbook, that is to say, by drawing the coins that represented the correct answer. Also, participants were asked to use pens, so that in the event of corrections, the students' first attempts would be visible. In the case of corrected answers, we counted these initial attempts as answers too. Finally, the students were asked to write down whether they encountered any difficulties in solving the problems and what kinds of difficulties these were, while they were also asked to comment on the specific problems in relation to the didactic objective set in the textbook's particular section.

- **How did they pay?**







	Bought	Change received	Draw how much money was given
Γαβριέλα			
Παντελής			
Μάρω			

Figure 1. *The problems used in the study (Kargiotakis et al. 2006a, vol. 1, p. 45).*

4. Results

The commercial transactions (the buying of toys) those appear in the problems act as a pretext for introducing children to the mathematical operations of adding and subtracting, and familiarising them with the use of coins. This is an approach to school mathematics that utilises cultural knowledge. The “realistic” problems commented upon here ask pupils to participate in public practices. However, the possibility of pupils responding to these problems presupposes an ability to adequately use the mathematics they are required to build (operations of addition and subtraction). As Dowling (2001) points out, this is an *ex ante* construct of mathematics as a necessary condition for the optimum participation of pupils in public practices, in this case financial practices, a fact which attempts to create their “myth of participation” (Dowling, 2001, p. 406) in said transactions.

The students' solutions appear on the next table in the form requested, i.e. drawings. Given that the mathematical content of problems was easy for these particular research subjects, we decided to include in our count the students' “corrected” answers, since they reflect their first thoughts towards solving the problems and provide useful indications regarding their hesitation and reasoning in the course of arriving at a solution.

Table. Categorisation of the answers, as they were presented in the form of drawings (The asterisk [*] shows how many subjects gave more than one solution in each case.)

	Suggested Solutions		N
1st problem	I	$\textcircled{2} \textcircled{2}$	18**
	II	$\textcircled{1} \textcircled{1} \textcircled{1} \textcircled{1}$	8**
	III	$\textcircled{1}$	1*
	IV	$\textcircled{1} \textcircled{1}$	2*
* In the 1st problem, one student gives solutions I and II, while another gives solutions III and IV.			
2nd problem	I	$\textcircled{2} \textcircled{2} \textcircled{2}$	18**
	II	$\textcircled{1} \textcircled{1} \textcircled{1} \textcircled{1} \textcircled{1} \textcircled{1}$	4**
	III	$\boxed{5} \textcircled{1}$	3*
	IV	$\boxed{10} \boxed{5}$	1
	VI	$\textcircled{1} \textcircled{1} \textcircled{1} \textcircled{1} \textcircled{1} \textcircled{50}$	1*
	VII	$\textcircled{1} \textcircled{1} \textcircled{1}$	1*
	VIII	$\textcircled{2} \textcircled{2} \textcircled{1} \textcircled{1}$	1
	IX	$\textcircled{2} \textcircled{2} \textcircled{2} \textcircled{2} \textcircled{2} \textcircled{2}$	1
* In the 2nd problem, one student gives solutions I, II and VII, one gives solutions I and VIII, one gives solutions II and III, and one gives solutions III and VI.			
3rd problem	I	$\boxed{5}$	16****
	II	$\textcircled{2} \textcircled{2} \textcircled{1}$	5*
	III	$\textcircled{1} \textcircled{1} \textcircled{1} \textcircled{1} \textcircled{1}$	6****
* In the 3rd problem, four students give solutions I and III, while one of them also gives solution II.			

4.1. Difficulties in “reading” and comprehending the empirical context

In the first problem, for example, the meaning of the concepts of addition or subtraction is constructed as a mediating relation between the empirical frame of reference on the one hand and the signs/symbols on the other. Thus, the empirical context presented in Figure 2 consists of the object

with the price tag on it and the change (€1), while the signs/symbols are the subtraction operation ($4-3=1$) or addition ($3+1=4$).

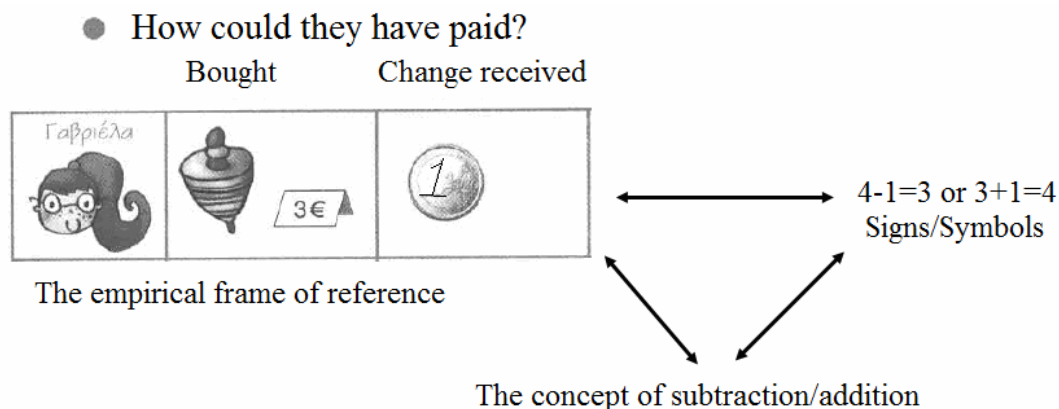


Figure 2. *The empirical framework of reference in the first problem*

In this particular example, the “correct” answer presupposes that the pupil:

- is acquainted with the concept of “change”;
- comprehends that the sum of the object’s value and the change is equal to the total amount of money he paid and is capable of correctly calculating the sum;
- chooses the form of the coins that correspond to the aforementioned sum so as not to include coins that exist in the change or coins of smaller value whose sum equals the value of the change;
- and, finally, it is presupposed that the pupil understands the social convention of not paying with coins that will then be returned as change.

Consequently, the empirical frame of reference introduces parameters related to financial transactions. Thus, in this problem, the students determine that the correct answer ought also to take into account the practices of a socially correct financial transaction rather than just any solution that would be considered correct by the standards of conventional arithmetic. In a “correct” financial transaction, in order to purchase an object priced at €3 and receive €1 change, one must pay with two €2 coins. From a mathematical viewpoint, however, these examples: $1+3=1+1+1+1$ or $1+3=1+1+1+0.50+0.50$ etc. are equally correct. In the example used here, the problem is distanced from this particular context, decontextualised and placed within a new context, which is dominated by the communication framework of the school class. This results in the problem being comprehended and interpreted as a mathematical problem (recontextualisation, Dowling 1998, 2001). This procedure leads to solutions that lie within the range of conventional arithmetic, as opposed to the initial empirical context which is dominated by social parameters and practices. The dysfunction of this particular frame of reference is indicated by the solutions suggested by the postgraduate students. A fair number of students (seven in the first problem, five in the second and eleven in the third, out of a total of twenty-seven students) provide answers that are considered “problematic” in a social context of transactions, even though most of them are mathematically correct, as in the case of the following student, whose answers are shown in Figure 3:

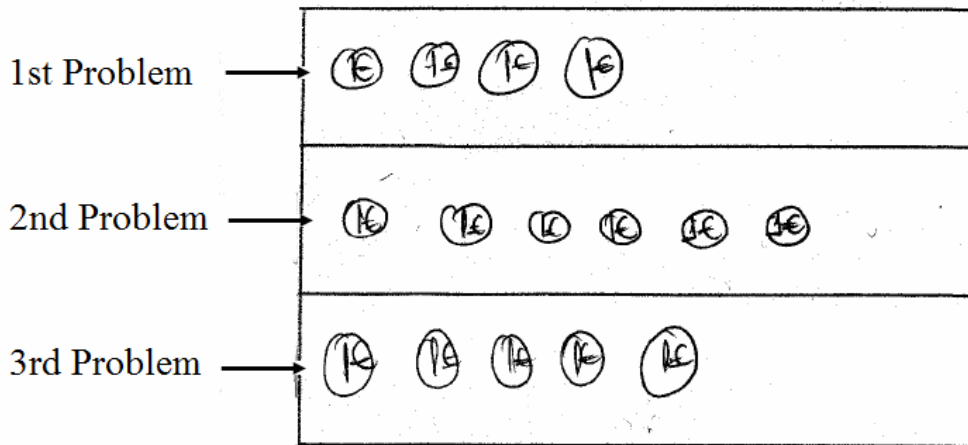


Figure 3. A mathematically correct solution but “problematic” in a social context of transactions

This particular postgraduate student comments on the problems as follows:

“The way the problems are given causes a little confusion in the beginning. At the same time, I think the operations are complicated for the 2nd grade, since euros and cents are involved in the same problem.”

Sometimes, the difficulties in comprehending the context lead to dilemmas; corrective interventions are then needed in order to achieve the expected solution, as is shown in Figure 4.

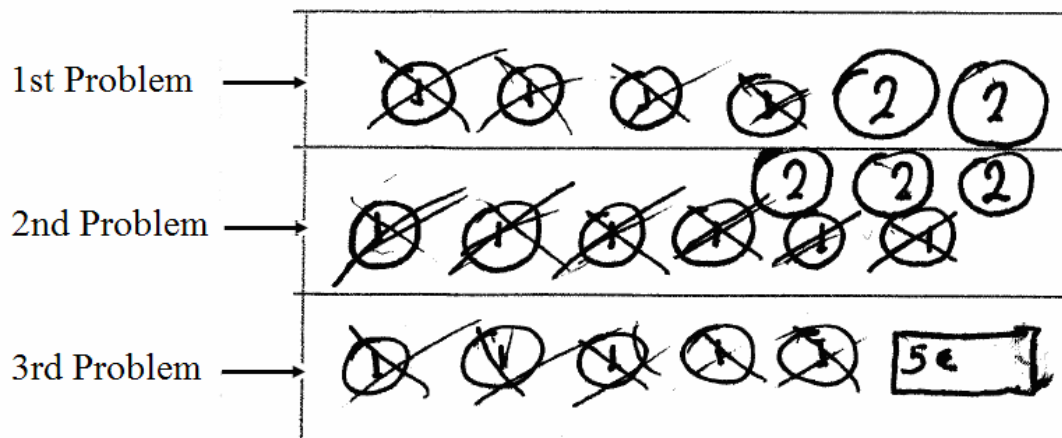


Figure 4: Difficulties in comprehending the context

The polysemy of the empirical context as well as the incompatibility between the context and the recontextualisation process are observed by the following student, whose answers are “correct”:

“In the first and second problem it is difficult for the children to figure out why they must pay with two €2 coins or three €1 coins respectively. That is also the case in the third problem with the €5 note. They may choose to answer in other ways, which may be correct from a mathematical point of view, but may not make sense in an actual situation.”

4.2. Difficulties in solving the problems

The postgraduate student who gave the solution seen above in Figure 4 describes the difficulties in comprehending as follows:

“I consider the information provided to the pupil to be insufficient. The pupil might easily assume that the value written in the first column (next to each object) is the amount of money paid and not the object’s price. [...] Moreover, no instructions are provided on how to use the coins.”

Certain students (three in the first problem and four in the second one, see Table) give wrong answers. This fact should be underscored, if we take into account the scientific profile of the students who were invited to solve problems intended for seven-year-old students.

In the case of one student’s mistake (Figure 5), we see that the suggested solution comes about as the result of the operation: $1+x=3$, i.e. $1+2=3$. Indeed, wishing to explain the problem’s difficulty, the student notes:

“It’s hard for pupils to think backwards and to add the price of a product to the change in order to come up with the money given.”

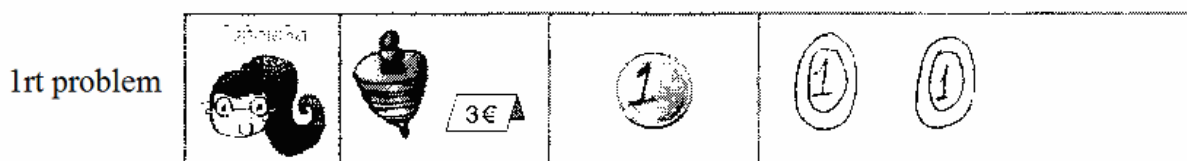


Figure 5. An erroneous “reading” of the context

The previous way of solving the problem seems to have also been chosen by another student, although in the end he gives the correct answer. He remarks:

“...The first thing that came to mind, mechanically, was to subtract $3-1$ and not to add $3+1$ to figure out how much money Gabriella gave.”

Here, the student’s approach may be due to the verbal information that accompanies the illustrations of the problems, since, as he remarks:

“The pupil might easily assume that the price that appears in the first column (next to each object) is the amount of money paid and not the price of the object...”

Another student, who gives correct and “anticipated” solutions, comments on the difficulty of the first problem:

“Yes, I encountered some difficulty, because if Gabriella [the girl in the illustration of the first problem] bought a spinning top, I assumed she would use 1 euro coins. But if she used 1 euro coins, why would she give the shop owner €4 only to get one back? Then I thought maybe she didn’t just buy one top, but three. So if she’d given a €10 note, she would have got €1 back in change...”

Similarly, the difficulties in “reading” the context are underscored by another student, who notes:

“The problem doesn’t clarify how many of each toy each child bought. So if I suppose that Pantelis [the boy in the illustration of the second problem] bought one car, he couldn’t have given a €5 note and a €1 coin to be given back the €1 coin plus 50 cents. I tend to think he bought three cars, in order for the change to make sense...”

As we see in the reasoning behind the proposed solution (see Figure 6), while the solution is mathematically correct, the particular “reading” of the context should be considered fanciful, since, as we have emphasised, these problems are intended for 2nd grade primary school pupils.

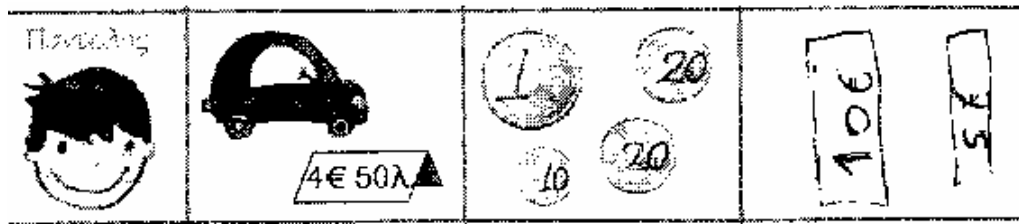


Figure 6. A fanciful interpretation of the context, based on the purchase of three cars

In the case of another postgraduate student, shown in Figure 7, an incorrect solution is given to the first problem, while various possible solutions are suggested for the second and third problem. This student remarks:

“The question is not formulated clearly. The first problem initially confused me, until I figured out the addition. I don’t know how easy it might be for a child in the 2nd grade to know the coins and their denominations and use them to perform addition and subtraction.”

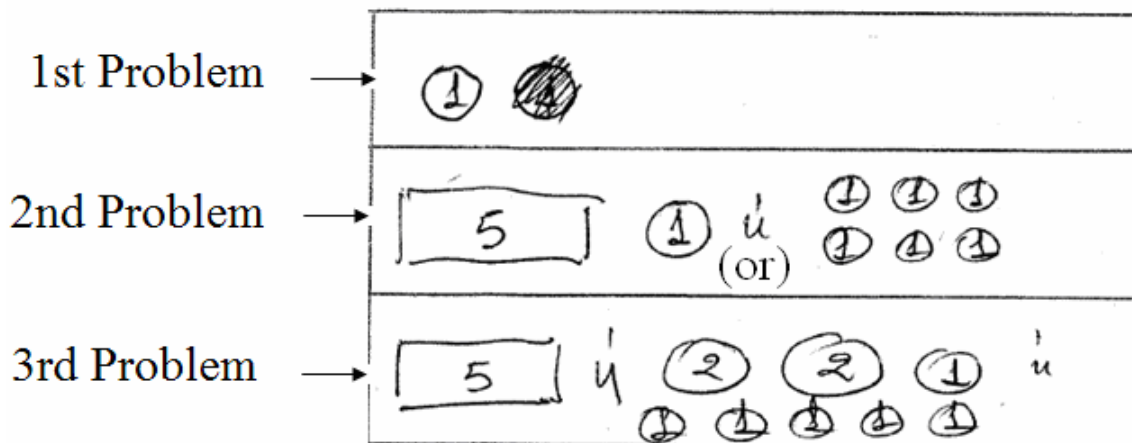


Figure 7. Another case in which the given empirical context is not taken into account

5. Conclusion and discussion

This research focused on the study of a selection of problems drawn from the mathematics textbook of the 2nd grade of Greek primary school, and specifically on the reference context of each of these problems. Our criterion for choosing these problems was to what extent they could generate commentary on aspects of their reference context.

In the theoretical part of the research we put forward approaches that stress the need for suitable reference contexts, a fact which can contribute to shaping mathematical knowledge on the part of pupils in the first primary school grades. In particular, emphasis was placed on the utilisation of data from the pupils’ everyday experiences, i.e., the kind of knowledge that B. Bernstein terms the horizontal discourse (1999).

From a semiotic perspective, which takes into account the cultural context within which learning takes place (Radford, 2006), the pupils’ interpretation of the reference contexts is a culturally mediated process, in the sense that pupils interpret the particular problems in terms suitable to the science of mathematics. Thus, the importance of parameters such as the financial transactions in our research is diminished and ignored. But, within the context of a realistic financial transaction, problems similar to those that occurred in our research demand a different approach.

Locating technical imperfections in the structure of these particular problems, the existence of which became clear in the remarks and the solutions of the postgraduate students/teachers of our sample, can create difficulties in their didactic management on the part of the teachers who are asked to teach them in the classroom. These difficulties lay mainly in the unfortunate choice of the empirical reference contexts, which distance themselves from the horizontal discourse of everyday knowledge, since they utilise cultural knowledge which is complex for young pupils, even though they have to do with the seemingly ordinary act of buying a toy.

Indeed, the reference context of the selected problems is encompassed by an ostensible “transparency” in regard to its “reading” and interpretation, which is based on the “realism” of the transactions. Pedagogical practice aims at activating the pupils’ interest and accepting their consent in tackling the mathematical content of the problems. But it seems that the effort to recontextualise and restructure everyday practices through the viewpoint of school mathematics, such as the problems selected by this research, creates a virtual reality. It appears that the content of mathematical problems is not always compatible with the daily practical activities of seven-year-old pupils or with their ability to handle transactions such as those described in particular problems. This fact was shown in the solutions proposed by the postgraduate students taking part in the research, in which we often observed a hesitation regarding the correctness of the proposed solutions. This hesitation was evident in their work sheets, where solutions were crossed out or attempts made to correct initial solutions. Moreover, if the frequency of erroneous solutions is not numerically high, it can nevertheless be considered significant, given that the particular problems are intended for seven-year-old pupils. The difficulties in interpreting the reference context discussed here result in the possible refutation of their desired pedagogical role, since they might lead to confusion and mistakes, as was the case in our research.

In concluding the commentary on the findings of this research, we feel we should underscore the special research interest of implementing this particular curriculum on the microlevel of the classroom and of observing didactic management methods of empirical reference contexts that display particularities, such as the contexts explored by this research.

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