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PRESERVICE PRIMARY SCHOOL TEACHERS' ELEMENTARY GEOMETRY KNOWLEDGE

Iuliana Marchis

Abstract: Geometrical notions and properties occur in real-world problems, thus Geometry has an important place in school Mathematics curricula. Primary school curricula lays the foundation of Geometry knowledge, pupils learn Geometry notions and properties by exploring their environment. Thus it is very important that primary school teachers have a good base in elementary Geometry. The aim of this paper is to present a research on how pre-service primary school teachers master elementary Geometry notions and properties related with some basic shapes and solids. The research shows that there are students, who can't recognize basic geometrical shapes or solids. Two third of the students can't define correctly basic geometrical shapes: they don't know the correct properties of the shapes; they know the properties of the shapes, but they repeat some properties in the definition or they miss some properties from the definition. As regarding geometrical solids, more than one third of the students couldn't draw the correct two-dimensional representation and one third didn't know how to draw the net of them.

Keywords: mathematics education, geometrical shapes, van Hiele levels of mental development in Geometry

1. Introduction

Geometry has an important place in school Mathematics curricula. It develops students' spatial ability, logical reasoning skills (French, 2004), and the ability to solve real-world problems in which geometrical terminology and properties occur (Jones & Mooney, 2003; Presmeg, 2006).

For a success in learning Geometry the understanding of geometrical concepts is essential. Elementary pre-service teahers have difficulties in understanding geometry concepts (Mayberry, 1981; Mason & Schell, 1988; Gutierrez & Jaime, 1999; Cunningham & Roberts, 2010; Kabaca, Karadag & Aktumen, 2011).

The aim of this paper is to present a research on how pre-service primary school teachers master elementary Geometry notions and properties as some basic shapes and solids.

2. Theoretical background

Spatial ability

Spatial ability is very important in many aspects of our life. Spatial ability is a set of abilities that includes "capacities to perceive the visual world accurately" (Gardner, 1993, p. 173). Linn & Petersen (1985) defined spatial ability as mental processes which are used in perceiving, storing, recalling, creating, arranging and making related spatial images. Thurstone (1950) divided spatial ability into three elements: spatial relations, visualization, and orientation. Spatial relations is the ability to recognize the relations between objects in the space. Spatial visualization is the ability to "perform imagined movements of objects in two-dimensional and three-dimensional space" (Clements & Battista, 1992, p. 444). Spatial orientation is the ability to recognize geometric shapes from different positions (Thurstone, 1950). Maier (1996) identified five factors of spatial ability: spatial perception, visualization, mental rotation, spatial relations and spatial orientation. Spatial perception is the ability to recognize the horizontal and the vertical. Mental rotation is the ability to rotate two- or three-dimensional figures.

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Spatial visualization ability is strongly related with students' Geometry achievement (Battista & Clements, 1991; Clements et al., 1997; Capraro, 2001).

Hiele's levels of mental development in Geometry

Van Hiele (1986) identified five levels of mental development in Geometry:

Level 1 (visualization): students can recognize a geometric object or concept based on a prototypical example. They can name and identify common geometric shapes (Prevost, 1985). For example, they recognize geometric shapes which are in a standard orientation.

Level 2 (analysis): learners can recognize a geometric shape based on its properties. Properties are not yet ordered at this level. For example, they might state, that a square is not a rectangle, a rectangle is not a parallelogram.

Level 3 (abstraction): students identify class inclusion of shapes. For example, they recognize that all squares are rectangles, but not all rectangles are squares. Students can give definitions; they recognize how a definition identifies clearly a notion.

Level 4 (deduction): learners understand the importance of proofs and they can construct geometric proofs. They are also conscious that a proof can be done in more than one way (Crowley, 1987).

Level 5 (rigor): learners understand the relations between geometrical concepts and they can see them in an abstract system. At this level students can study non-Euclidean geometries (Crowley, 1987).

Students' misconceptions in geometry

Concept definition is "a form of words used to specify that concept" (Tall & Vinner, 1981, p. 152). Formal concept definition generates a personal concept image. Students' prior experiences with the geometrical concept embody the concept image (Vinner & Hershkowitz, 1980). In some students, this concept image may not develop; in others, it may not, be coherently related to the formal concept definition. These misconceptions have to be addressed during instruction in order to make the student to contemplate where the conflict between the formal definition and their own concept image occurs. In the following let see some common misconceptions.

Many students have problems in recognizing different geometrical shapes in non-standard orientation, for example, a square is not a square if its base is not horizontal (Mayberry, 1983; Clements & Battista, 1992).

Many students have difficulties to perceive class inclusions of shapes (Mayberry, 1983; Feza & Webb, 2005), for example, they might think, that a square is not rectangle (Marchis, 2008), a square is not rhombus, a rectangle is not parallelogram (Clements & Battista, 1992).

Some students can't recognize geometrical solids or/and they can't draw the net of these solids (Pittalis, Mousoulides & Christou, 2010).

Geometrical shapes and solids

In this section we give the definition of some geometrical shapes which will be included in the research.

A **parallelogram** is a simple quadrilateral with two pairs of parallel sides.

A **rectangle** is a simple quadrilateral with four right angles. We can define a rectangle based on a parallelogram: a rectangle is a parallelogram with at least one right angle.

A **rhomb** is a quadrilateral with four sides of equal length. We can define a rhomb based on a parallelogram: a rhomb is a parallelogram in which at least two consecutive sides are equal in length.

A **square** is a simple quadrilateral with four equal sides and four equal angles. It can be defined based on parallelogram, rectangle of rhomb. A square is a parallelogram with one right angle and two adjacent equal sides. A square is a rectangle with two adjacent equal sides. A square is a rhombus with all angles equal or a square is a rhombus with at least one right angle.

As regarding geometrical solids, those studied are tetrahedron, square pyramid, triangular and tetragonal prism.

A **pyramid** is a polyhedron formed by connecting a polygonal base and a point. If the base is a triangle we got a **tetrahedron**. If the base is a square we got a **square pyramid**.

A **prism** is a polyhedron with a n-sided polygonal base, a translated copy of it to another plane, and n other faces joining corresponding sides of the two bases. If the bases are triangles, we speak about **triangular prism**. If the bases are quadrilaterals, we speak about **tetragonal prism**. If all the faces of a tetragonal prism are squares, we get a **cube.** Thus a cube is a particular tetragonal prism.

3. Research

Research design

The goal of the research is to evaluate pre-service primary school teachers' basic knowledge regarding geometrical shapes and solids.

The research was done during June 2012.

The sample is formed from a group of 36 students studying for a degree in preschool and primary school education. They were third (final) year students. Only one male respondent was included in the sample, this percentage reflects the educational reality.

The research tool is a set of problems related with plane and space geometrical forms. In the first part students have to give the definition of a parallelogram, rectangle, rhomb, and square; and describe their properties. In the second part students have to draw the following geometrical solids and their net: tetrahedron, square pyramid, triangular, and tetragonal prism.

Results and discussion

13 students from 36 gave correct definitions for the **geometrical shapes**. In the following we present some of the students' definitions for basic geometrical shapes.

First of all, let see how they define a **rectangle.**

"The rectangle is a geometrical shape, which has four sides, two opposite sides are parallel and equal, its angles are right angles, its diagonals are equal and they intersect each other in the center." This is not a definition; it is a list of properties. The students doesn't know how to give a definition.

"The rectangle is a parallelogram whose opposite sides are parallel and equal." The opposite sides of a parallelogram are parallel and equal, thus these properties are repeated and other property important for a rectangle (the angles of a rectangle are equal) is omitted.

"The rectangle is a parallelogram whose opposite sides are parallel and its angles are right angles." This definition describes a rectangle, but some properties are repeated: the opposite sides of a parallelogram are parallel, so we don't need to include this property in the definition.

As regarding a **rhomb**, some students also gave incorrect definition.

"The rhomb is a parallelogram whose opposite sides are equal." The opposite sides of a parallelogram are equal, so this definition repeats two times the same property and omit others. In this definition the student is aware of the fact that the rhomb is a particular parallelogram.

"The rhomb is a square whose opposite sides are parallel and equal." A square is a parallelogram, so it's opposite sides are parallel and equal. Thus these properties are repeated. The square is a particular rhomb, thus we can't define a rhomb based on a square (only vice versa).

Even pupils meet the notion of the **square** in preschool; some of the preservice primary school teachers can't define it correctly.

"The square is a rectangle whose sides are equal and parallel." The opposite sides of a rectangle are equal and parallel (as a rectangle is a parallelogram), thus this information is repeated in this definition. An important property of the square is not included: two adjacent sides are equal.

"The square is a quadrilateral whose sides are equal." This definition point out one property of a square: all the sides have equal length. But rhomb also has all the sides equal. Another important property is missing in order to describe a square: at least one of the angles is right angle.

"The square is a rectangle whose sides are equal and adjacent sides are perpendicular to each other." This definition describes a square but it contains repeated information: the adjacent sides of a rectangle are perpendicular.

We could observe that students are not familiar with formulating definitions, they don't aware with the fact that in a definition we include the minimum amount of properties which clearly describe the notion. Some of the definition, even if they are correct, contains repeated information. For example, in "The square is a rectangle whose sides are equal and adjacent sides are perpendicular to each other." the property that adjacent sides are perpendicular to each other is repeated.

In other cases the definitions given by them are missing important properties of the definied notion. For example, in "The square is a quadrilateral whose sides are equal." the fact, that at least one angle is right angle should be pointed out.

Some of the definitions are incorrect due to the fact that students are not aware on the class inclusion of the shapes. For example, in "The rhomb is a square whose opposite sides are parallel and equal." the rhomb is defined based on a square.

We could conclude, that two third of the students are only on level 2 (analysis) on Van Hiele's levels of mental development in Geometry.

When representing geometrical solids, the plane representation is the most frequently used, but this requeries considerable conventionalizing (Parzysz, 1988). Thus students usually have difficulties with this representation (Ben-Chaim, Lappan & Houang, 1989; Gutierrez, 1992; Pittalis, Mousoulides & Christou, 2010). The results of our research show that 21 students from 36 draw correctly the two-dimensional representation of the required geometrical solids.

The most common mistake when drawing geometrical solids is, that students are not aware on the fact that some of the edges can't be seen (Figure 1). 12 students from 36 gave two-dimensional representation in which is not marking the invisible sides. One student gave a very interesting solid instead of a square pyramid (see Figure 2). 5 students draw cube for tetragonal prism, these students are not aware on the fact that usually when we have to give some geometrical shapes or solids we don't choose partivular cases.

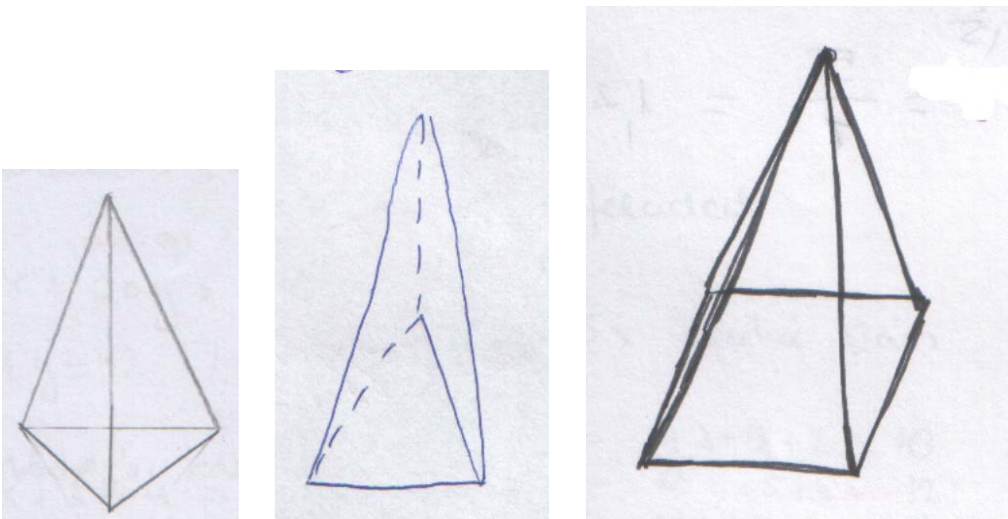

Figure 1. Students are not aware thet some of the edges can't be seen

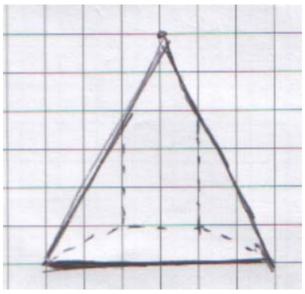

Figure 2. Student's square pyramid

Net construction requires focusing on the components of the solid. 24 students from 36 gave the correct net of the required solids. 8 students from 36 didn't draw anything for the net; maybe they didn't know the notion of the net of a solid. One student drew the body diagonal instead of the net (see Figure 3). In Hungarian language the net of a solid is "testháló", and the body diagonal of a solid is "testátló", so these two notions are very similar, this could be the origin of this confusion.

Other students didn't draw the net of the solid they gave. In Figure 4 we could observe, that the student drew a cube, but the net is composed from rectangles, so it is not the net of a cube. In Figure 5 the solid is a square pyramid and the net is belonging to a tetrahedron. This inconcordance between the solid and the net shows, that students can't imagine in their mind the three-dimensional object and the fact how they open it in order to get the net; they only try to draw something from their memory.

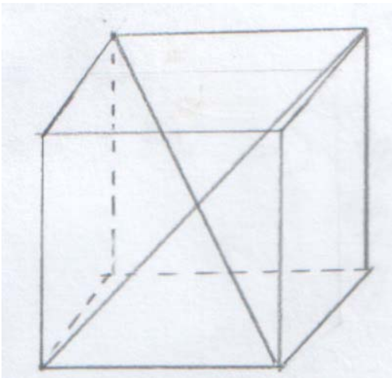

Figure 3. Student draw body diagonal instead of the net of the solid

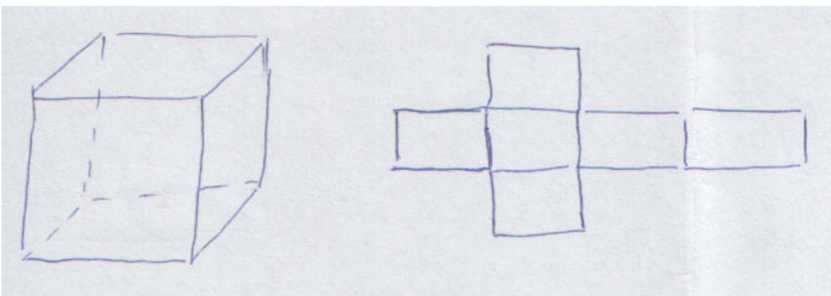

Figure 4. The net is not in correspondence with the given solid

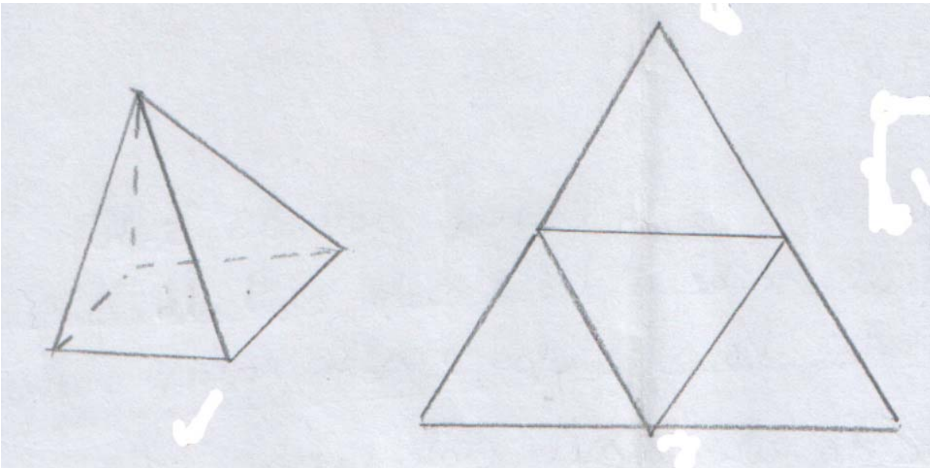

Figure 5. The net is not in correspondence with the given solid

4. Conclusions

The results show that pre-service preschool and primary school teachers are lackink elementary Geometry knowledge.

Two third of the students can't define correctly basic geometrical shapes for different reasons:

- they can't recognize the geometrical shape;
- they don't know the correct properties of the shapes;
- they know the properties of the shapes, but they repeat some properties in the definition;
- they know the properties of the shapes, but they miss some properties from the definition.

As regarding geometrical solids, more than one third of the students couldn't draw the correct two-dimensional representation and one third didn't know how to draw the net of them.

Thus pre-service preschool and primary school teachers' Geometry knowledge has to be enriched and their geometrical thinking and spatial ability have to be developed.

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Author

Iuliana Marchis, Babes-Bolyai University, Cluj-Napoca, Romania, e-mail: iuliana.marchis@ubbcluj.ro