EDUCATIONAL EXPERIENCES ABOUT USING DIFFERENT COMPUTER PROGRAMS IN CALCULUS COURSES

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Abstract: Traditionally the mathematical *Calculus* course starts from the first semester at the University of Kaposvár. On the basis of our experiences it is too difficult or abstract for our students to pick up the elements of this subject. Among others this motivated us to use computerized methods for demonstrations, visualization and better understanding of the concepts (monotonity, extreme values, etc.) of the curriculum. We decided for programs Excel, Geogebra and Euler3D as these are available and easy to use. Students draw graphs of functions and sequences with Excel or/and Geogebra and recognize their properties, and also use Excel Solver and Euler3d programs to solve and demonstrate linear programming problems. Either the graphic representing of functions or the visualization of linear programming exercises can be solved easily with the help of computer programs unlike the paper pencil method. During the resolving of analytical exercises the emphasis goes from the symbolic level that was defined by Bruner to the representational level of the demonstration or graphs. In this paper we give examples from students' tasks and opinions.

Key words: mathematical education using computer, representation levels, calculus

1. Introduction

Our experiences show that teaching-learning process is damaged on the different levels of mathematics education. In my opinion the problem is that in teaching-learning process the foundations are left for higher education, but this way the acquisition of other subjects is hindered too, because the "laying of foundations" is not finished yet. How can we make up for these difficulties in higher education? This topic is important for our students, because of Calculus is a basic subject and they have to get acquainted with functional operations in order to be able to describe economic processes with the help of functions. I am looking for computer methods and programs for teaching analysis, which are very useful in drawing graphs of functions, recognizing their properties.

Before starting their studies, students are tested in mathematics. Questions in the test are about number- and function-abstraction and about model creation. We reveal their deficiencies based on their solutions.

We offer an optional subject to the students. It is called *Methods of Mathematics Using the Computer*. This course goes in parallel with *Mathematics I (Calculus)*.

The subject has a threefold aim:

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- the development and conditioning of the bases;
- to link it closely with higher mathematics;
- to link it with the use of computers.

Cognitive mathematics learning objectives were used during the training course (Bloom,1972; Varga 1973):

- **Understanding** (relationship insight, understanding the statement, repetition, thought up, the transition between the use of representations)
- **Knowledge-action services** (routine, solution algorithms, skills)
- **Construction** (problem formulation, generalization, each possible solution)
- **Assessment** (whether the statement is true, mutatis mutandis, whether the data are sufficient; thought fairness, rationality, practicality)

I have been dealing with Bruner's representational theory and I try to adapt it to my research. Bruner examined how people represent and store the information arriving from the external world. Almost thirty years ago, when current graphic computer environments were not even a dream, Bruner (1966) distinguished three different modes of mental representation – the sensori-motor, the iconic and the symbolic. In his essay "Patterns of Growth" he wrote:

> What does it mean to translate experience into a model of the world. Let me suggest there are probably three ways in which human beings accomplish this feat. The first is through action. […] There is a second system of representation that depends upon visual or other sensory organization and upon the use of summarizing images. […]

> We have come to talk about the first form of representation as enactive, the second is iconic. […] Finally, there is a representation in words or language. Its hallmark is that it is symbolic in nature. Bruner, 1966, pp. 10–11.
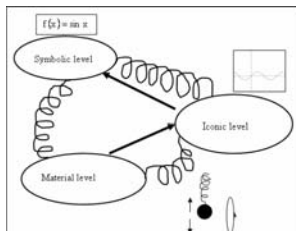
The three representation methods take part in each phase of the teaching process. To my mind the visual education is very important, that's why I try to provide everyday, lifelike illustrations to help the acquisition of the material.

Figure 1. Bruner's three modes of representation

Although this claim is far-reaching, it has proved to be able to explain certain sophisticated logical ideas in, say, a simple visual form. However, different representations use different forms of knowledge which has advantages and hidden difficulties. In figure 1 the bold lines show Bruner's proposed sequence of growth, together with other connections between the systems. For instance, the symbolic system passes written communication to the enactive system for writing and typing, the iconic system passes its drawing actions to the enactive system and there are many links between iconic and symbolic in mathematics, for example between symbolic functions and graphs (Tall, 1994). There is flexibility between the different levels: the easier the transition between the levels, the more effective the representational thinking and problem-solving are. The use of computers can improve flexibility in the transition between the representation levels. Computer programs help students to draw the graph of a function. We had to look for programs which are available for everyone, so our choice fell on Excel and GeoGebra.
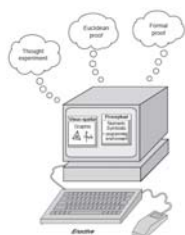

Figure 2. Using the computer to provide visuo-spatial and proceptual support for various levels of mathematical thinking (Tall,1994)

2. Drawing the graph of a function using Excel

In order to be able to draw the graph of a function, it is necessary to find the extreme value points, monotonic sections, the curvature of the function, and the inflection points by a means of analysis. Students are able to find these characteristics and to determine the part of the domain of the function where it should be plotted. However, it still presents difficulties for them to draw the graph of the function, that's why I started to look for programs that help representation.
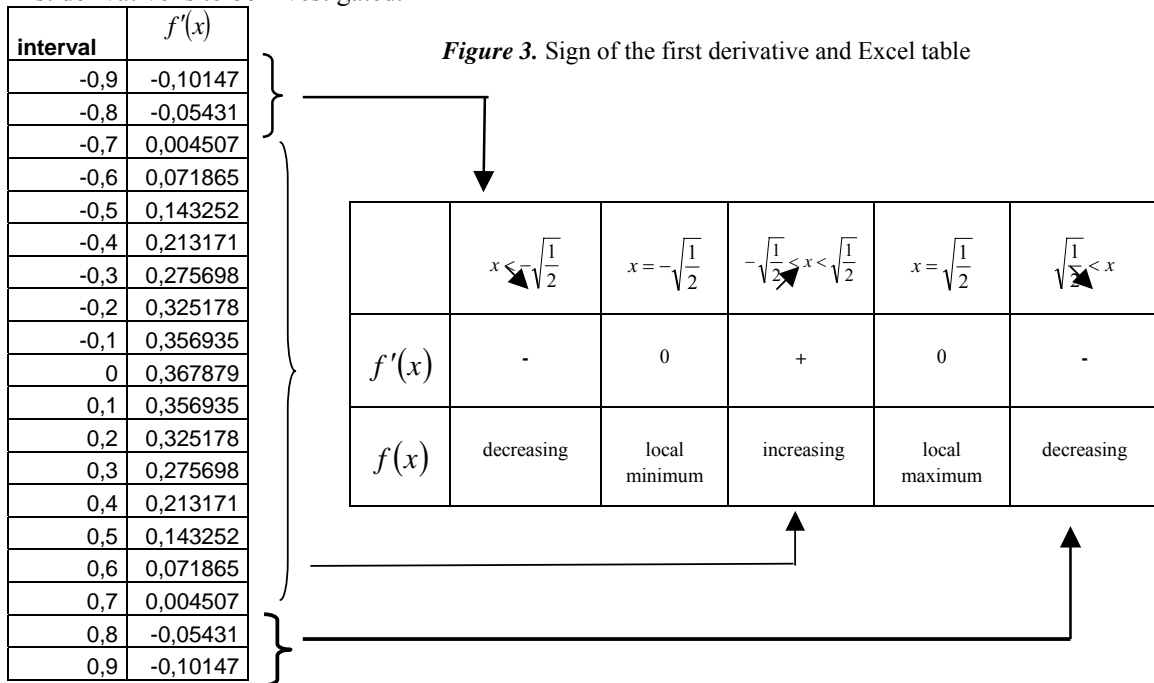
For example one task was:

Determine the first and second derivative. They have to determine the intervals where the characteristics of the function change:

$$f(x) = \frac{x}{e^{x^2+1}}$$

We define the points necessary for drawing the graph of the function with the means of mathematical analysis:

- zero location: $f(x) = 0 \Rightarrow$ x=0

- possible locations of extreme values: $f'(x) = 0$ $f'(x) = \frac{1-2x^2}{e^{x^2+1}}$ $\Rightarrow$ $x_{1,2} = \pm\sqrt{\frac{1}{2}} \approx \pm 0{,}707$

The sign table made by Excel: The received zero points determine the intervals where the sign of the first derivative is to be investigated.

interval	$f'(x)$
-0,9	-0,10147
-0,8	-0,05431
-0,7	0,004507
-0,6	0,071865
-0,5	0,143252
-0,4	0,213171
-0,3	0,275698
-0,2	0,325178
-0,1	0,356935
0	0,367879
0,1	0,356935
0,2	0,325178
0,3	0,275698
0,4	0,213171
0,5	0,143252
0,6	0,071865
0,7	0,004507
0,8	-0,05431
0,9	-0,10147

Figure 3. Sign of the first derivative and Excel table

	$x < -\sqrt{\frac{1}{2}}$	$x = -\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}} < x < \sqrt{\frac{1}{2}}$	$x = \sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}} < x$
$f'(x)$	-	0	+	0	-
$f(x)$	decreasing	local minimum	increasing	local maximum	decreasing

It would be sufficient to substitute only one element in each interval because of the continuity of the derivative function. The reason why there are so many elements substituted is that students should actually verify the correctness of certain elements of the sign board.

The second derivative of the function is to be determined.

- possible locations of the inflection point : $f''(x) = 0$

- in a further simplified form: $f''(x) = \frac{-6x+4x^2}{e^{x^2+1}}$

- the zero locations of the second derivative: $x_{1,2} = \pm\sqrt{\frac{3}{2}}$

The second derivative signs table shows the curvature of the original function.

interval	$f''(x)$
-1,3	-0,06707
-1,2	0,025102
-0,1	0,217075
0	0
0,1	-0,21707
1,2	-0,0251
1,3	0,067066

$f''(x)$	$f(x)$	interval
-	∩	$x < -\sqrt{\dfrac{3}{2}}$
0	inflection point	$x = -\sqrt{\dfrac{3}{2}}$
+	∪	$-\sqrt{\dfrac{3}{2}} < x < 0$
0	inflection point	$x = 0$
-	∩	$0 < x < \sqrt{\dfrac{3}{2}}$
0	inflection point	$x = \sqrt{\dfrac{3}{2}}$
+	∪	$\sqrt{\dfrac{3}{2}} < x$

Figure 4. Sign of the second derivative and Excel table

Most students are able to find the elementary characteristics of a function. However, only few if any are able to illustrate those characteristics which can be obtained with the help of differential calculus. Excel provided help in this problem. I discovered that students enjoyed using the computer for checking, after we had solved the tasks in a traditional way.
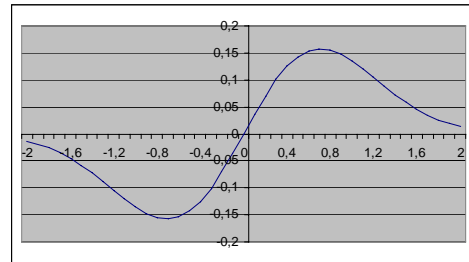

Figure 5. Graph of the f(x) function in Excel

3. GeoGebra in the education of analysis

It was this semester when I used GeoGebra for the first time at Calculus class work. Why GeoGebra? We looked for such a program which is free of charge and which can be downloaded from Internet. GeoGebra has an easy-to-use interface, built-in mathematical commands, so it did not cause any problems to use.

3.1. Function Analysis

Let's see the function defined by the following formula.

$$f(x) = \frac{x^2 + x - 1}{e^x}$$

We define the points necessary for drawing the graph of the function with the means of mathematical analysis:

- zero location : $f(x) = 0 \implies x^2 + x - 1 = 0 \implies x_{1,2} = \dfrac{-1 \pm \sqrt{5}}{2}$

- possible locations of extreme values: $f'(x) = 0 \implies f'(x) = \dfrac{(2x+1)e^x - (x^2 + x - 1)e^x}{(e^x)^2}$

How does GeoGebra help?

The relation is as follows: $(e^x)^2$

$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

Free objects= szabad alakzatok

Dependent objects= függő alakzatok

In the algebraic box you can check if the derivation is correct, with the help of GeoGebra we get the non-simplified derivative function.

- in a further simplified form: $f'(x) = \dfrac{-x^2 + x + 2}{e^x}$,

- the zero locations of the first derivative: $x_{1,2} = \langle \begin{matrix} -1 \\ 2 \end{matrix}$

sign chart: the relationship of the function and its first derivative

	$x < -1$	$x = -1$	$-1 < x < 2$	$x = 2$	$2 < x$
$f'(x)$	-	0	+	0	-
$f(x)$	decreasing ↘	local minimum	increasing ↗	local maximum	decreasing ↘
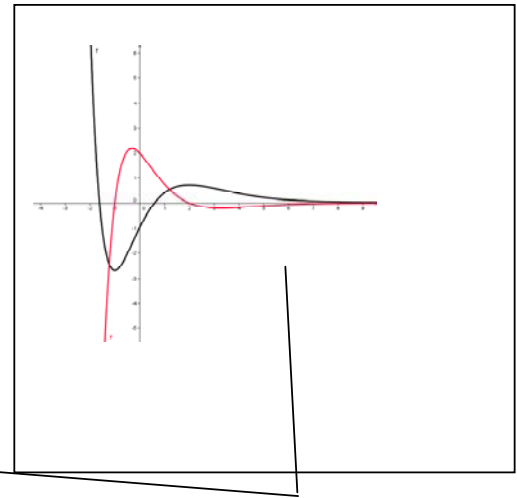
The sign chart can be checked knowing the graphs of the functions.

- possible locations of the inflection point : $f''(x) = 0 \Rightarrow$

$$f''(x) = \frac{(2x+1)e^x - \left(-x^2 + x + 2\right)e^x}{\left(e^x\right)^2} \Rightarrow$$ in a further simplified form: $f''(x) = \dfrac{x^2 - 3x - 1}{e^x}$

- the zero locations of the second derivative: $x_{1,2} = \dfrac{3 \pm \sqrt{13}}{2}$

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Szabad alakzatok
    f(x) = (x² + x - 1) / e^x
Függő alakzatok
    f'(x) = (e^x (2 x + 1) - (x² + x - 1) e^x) / e^x²
    f''(x) = (e^x² (2 e^x + e^x (2 x + 1) - ((x² + x - 1) e^x + (2 x + 1) e^x)) - (e^x (2 x + 1) - (x² + x - 1) e^x) 2 e^x²) / e^x⁴
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On the one hand, GeoGebra defines the second derivative from the non-simplified form of the first one, which makes the algebraic form of the derivative function more complex in the case of a fraction. On the other hand, if we want to ask our students only about derivation and its steps without the simplified forms, they can profit from it in the checking process.

	$x < \frac{3-\sqrt{13}}{2}$	$x = \frac{3-\sqrt{13}}{2}$	$\frac{3-\sqrt{13}}{2} < x < \frac{3+\sqrt{13}}{2}$	$x = \frac{3+\sqrt{13}}{2}$	$\frac{3+\sqrt{13}}{2} < x$
$f''(x)$	+	0	-	0	+
$f(x)$	convex ∪	inflection point	concave ∩	inflection point	convex ∪
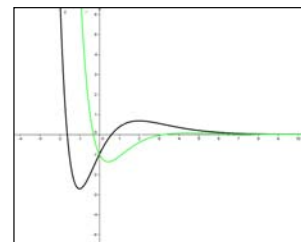
The function and its derivatives are represented in the same coordinate system, which helps students to deepen their knowledge. The sign of the first derivative function indicates if the original function is increasing or decreasing and gives the possible locations of extreme values. The sign of the second derivative function shows the curvature of the original function and the position of inflection.

Students can check the sign chart with the comparison of the second derivative of the function drawn by GeoGebra and the graph of the original function.

4. Conclusion

Students in economics have to know the traditional way to learn the processes of the bases of Calculus. Excel and GeoGebra programs are a supplement to this, and help verification and illustration. Our experience shows that students have the most difficulties with drawing the graph of a function. This is where computers can help.

Here is an opinion of a student:
For the solution of the two tasks we used Excel and GeoGebra programs, and the use of GeoGebra was found much faster and easier. On the one hand, it is not necessary to specify the intervals; on the other hand, it is not necessary to enter values. In addition, typing formulae is simpler. The representation and use of the functions show much better properties in GeoGebra, and they are more visible as well.

Literature

[1] Ambrus András (2004): *Bevezetés a matematika-didaktikába*, ELTE Budapest, 23-40.

[2] Ayub,AF.M.,Tengku Mohd Tengku Sembok,T.M.T., Luan,W.S. *Teaching and Learning Calculus Using Computer (2008)*, URL:
http://atcm.mathandtech.org/EP2008/papers_full/2412008_15028.pdf. [29.08.2009.]

[3] Chris Little (2009) *Differentiation in three easy, GeoGebra-style, lessons*, MSOR Connections Vol 9 No 2 May – July 2009

[4] David R. Green and Carol L. Robinson (2009) *Introducing GeoGebra to foundation year students*, MSOR Connections Vol 9 No 2 May – July 2009

[5] David Tall (1994) *A Versatile Theory of Visualisation and Symbolisation in Mathematics*, Plenary Presentation at the Commission Internationale pour l'Étude et l'Amélioration de l'Ensignement des Mathématiques, Toulouse, France, July 1994.

[6] Güyer, T.(2008) *Computer Algebra Systems as the Mathematics Teaching Tool*, World Applied Sciences Journal 3 (1): 132-139, 2008 ISSN 1818-4952

[7] Hohevarter,M., Hohenvarter, J., Keis, Lavicza: *Teaching and Learning Calculus with Free Dynamic Mathematics Software GeoGebra*, TSG 16: Research and development in the teaching and learning of calculus ICME 11, Monterrey, Mexico 2008

[8] Junaini,S. N. and Sidi,J. (2006) *Enteramath: interactive online mathematics teaching and learning through animations*, Proceeding of the International Conference on Distance, Collaborative and eLearning (DCEL 2005) 45 January 2006 Kuala Lumpur

[9] Ooi, A. (2007) *An Analysis of the Teaching of Mathematics in Undergraduate Engineering Courses*, Proceedings of the 2007 AaeE Conference, Melbourne